Internet Mathematical Olympiads

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Abstract
Modern Internet technologies open new possibilities in a wide spectrum of traditional methods, used in mathematical education. One of the areas, where these technologies can be efficiently used, is an organization of mathematical competitions. Contestants can stay in their schools or universities in different cities and even different countries and try to solve as many mathematical problems as possible and then submit their solutions to organizers through the Internet. Simple Internet technologies supply audio and video connection between participants and organizers in a time of the competitions.

Introduction
One of the main problems in the organization of National and International mathematical Olympiads is their expensiveness for potential participants. Team's organizer has to find a corresponding found or a sponsor, which can support a team, in order to bring his students to different cities or even different countries and to organize their accommodation there. Note also that all money collected by team's organizers is actually passed to tourism companies rather than the persons who prepared teams to competitions for many months.
Innovative ideas based on the use of Internet technologies can essentially change all this situation. The proposed model of Internet Mathematical Olympiad is very cheap and convenient for participants and organizers.

Description of Realization of Internet Mathematical Olympiad
Problems are posted on a dedicated area in the website for 3-4 hours. Contestants can stay in their schools or universities in different cities and even different countries and try to solve as many mathematical problems as possible and then submit their solutions to organizers through the Internet.
Organizers arrange a broadcast to all participants through standard programs. Participants and specialists, who are interested in the competition, can also watch the opening and closing ceremonies of the Internet Mathematical Olympiads. All this strengthens the effect of presence in order to influence the motivation of students to study non-standard mathematical problems. Our main idea is to attract as many students as possible for this activity, and one of our aims is for students to enjoy it.

To Lessen Negative Psychological Effects of Losers
In every mathematical competition, corresponding psychological problems can appear. Although we invest a lot of time and efforts in choosing suitable competitors, we find that students, who lose in the competition, lose their confidence. As a result, these students do not want to participate in future competitions and are left out of our organizing efforts instead of getting additional motivation in studying mathematics, which can lead even to corresponding psychological problems. In order to lessen negative effects of losers, we propose two ideas.
The first one is that only the names of the best contestants in the top of the winner's list are published. The second one is that the list of problems should be long enough.

Our main principle in choosing problems is that they have to be interesting to solve or at least to try to be solved by as great a number of students as possible.

Grading System

The grading system is based on the principle that the points for each problem are graded according to its rate, which is inversely proportional to the number of students who solved the problem correctly. As a result, simple problems are able to bring only a small number of points. This avoids essential influence of simple problems on the final result. It is a possible situation, when the absolute winner solved less problems correctly than several other contestants, but among the problems he/she solved were problems with a higher rating. This factor is an element of competition game, what gives students of schools or first year, who know less, a chance to win. That way they can prove their creativity in solving nonstandard problem.

Our Experience

We started Internet Mathematical Olympiads for Israeli students in 2006. Since 2007 our Internet Olympiad has become international. Students from 14 countries participated in our Olympiad in December 2008. More than 35000 enters to the site of our Olympiad demonstrates a wide interest of students in our project. As an example, how the problems of the Internet Mathematical Olympiad can be chosen, we present the problems from one of our competitions.

Problems of the Internet Math Olympiad, March 19, 2008

Problem 1.

Calculate the following limits:

a) $\lim_{n\to\infty}\sum_{k=n}^{2n}\frac{1}{k!}$, b) $\lim_{n\to\infty}\sum_{k=n}^{2n}\frac{1}{k}$.

Problem 2.

Prove that for any 9 interior points of a cube whose sides equal to 1, at least two of them can be chosen such that the distance between them does not exceed $\frac{\sqrt{3}}{2}$.

Problem 3.

a) Calculate the sum of the infinite series $\sum_{n=1}^{\infty}\frac{n}{(n+1)!}$,

b) Find the following limit: $\lim_{n\to\infty}\frac{1}{\sqrt{n}}\int_1^n \ln\left(1+\frac{1}{\sqrt{x}}\right)dx$.

Problem 4.

Prove that

$$\int_0^{\sqrt{2\pi}}\sin\left(x^2\right)dx > 0$$

Problem 5.

Prove that for any polynomial p(x) of the degree n and any point Q
the number of tangents to its graph which pass through the point Q does not exceed n.

Problem 6.

Let A,B,C,D be four distinct spheres in a space. Suppose the spheres A and B intersect along a circle which belongs to some plane P, the spheres B and C intersect along a circle which belongs to some plane Q, the spheres C and D intersect along a circle which belongs to some plane S and the spheres D and A intersect along a circle which belongs to some plane T. Prove that the planes P,Q,S and T are either parallel to the same line or have a common point.

Problem 7.

For a square matrix A denote

$$\sin A = A - \frac{A^3}{3!} + \frac{A^5}{5!} - \frac{A^7}{7!} + \frac{A^9}{9!} - ...$$

a) Prove, that if the matrix A is symmetric $A = A^T$, then all elements of the matrix $\sin A$ belongs to the segment [-1,1].
b) Is the above assertion true for non-symmetric matrix A?

Problem 8.

All the position of a cellular tape are numerated by the numbers 0,1,2,3,… and in some of them one or more game pieces can be placed. Our moves are determined by the following rules:

1) If in all of the positions whose numbers are $n \geq 1$ there is no more than 1 game piece in each, we add 2 game pieces into position number 1.

2) Otherwise, the position $n \geq 1$ with the maximal number from all the positions which have at least two game pieces is chosen, and then 2 game pieces are moved from this position in two opposite directions: one of them is moved from the chosen position n to the position n-k and another game piece is moved from the chosen position n to the position n+k, where k is an integer number $(1 \leq k \leq n)$. This number k can be chosen arbitrary for each move.

What is the maximal number of moves that can be made so that no game piece will be in the positions with numbers greater than 2008?

Problem 9.

A matrix A 2008×2008 is given. All its elements equal 0 or 1. Assume that every two lines differ from each other in a half of the positions. Prove that every two columns in this case also differ in a half of the positions.

Problem 10.

Let $\frac{\alpha_n}{\beta_n}$ be an irreducible fraction of the form $\frac{\alpha_n}{\beta_n} = \sum_{k=1}^{n} \frac{1}{k}$

Let us call the prime number p a good number if it is a divisor of α_n for a some n. Prove that the set of all good numbers p is infinite.

Note that the majority of these problems are based on the problems written by Prof. Alexei Kannel-Belov and Lev Radzivilovsky, coaches of the Israeli student team on mathematics.