

Mathematical Competitions for University Students

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Abstract

We present several possible forms of mathematical competitions for University students. One of them is Blitz Mathematical Olympiad. It is a team competition, when all teams receive the same problem and are allotted 10-15 minutes to come up with a solution. This cycle is repeated 6-8 times with different problems. Modern Internet technologies allow us to organize Blitz Mathematical Olympiads for the teams which are in different cities and even countries.

Introduction

Lecturers teaching mathematical courses for students, whose main specialization is not mathematics, can confirm that almost all their efforts and time (except lectures of course) are invested only on the weakest students in order to help them survive in academic institutes. Hardly anything is done for average or even relatively strong students, whose level cannot be defined as an excellent one, but they understand the basic things in mathematics and, in corresponding sense, they even like mathematics. In the system of high education there are almost no developed forms of the work with good students, although namely these students will become the kernel of specialists working in industry, education, hi-tech and even in science. Many enthusiastic lecturers tried to find such forms. Scientific circles for students and the institute Math Olympiads present only a short list of ideas in this area, but all of these forms require a lot of time and efforts of their organizers. However, only a very small part of students was involved in this activity. The scientific circles were very similar to additional courses and were usually stopped when the real exams approached. Only a few students participate in various Mathematical Olympiads.

Several Notes about Classical Mathematical Olympiads

Mathematical competitions among students have two main goals. The first goal is just like any other competition - to discover the strongest competitors. The second is to enhance interest in mathematics. The first aim is quite achievable, however the other goal, is far less attainable. Mathematical competitions in the classical form of the exam is not, in our opinion, the best way to incite the students' interest. Although we invest a lot of time and efforts in choosing suitable competitors, we find that students, who lose in the competition, lose their confidence. As a result, these students are reluctant to participate in future competitions and are left out of our organizing efforts instead of getting additional motivation in studying mathematics, which leads to serious psychological problems. If we want a mass of students to participate in mathematical competitions we have to choose their forms in a way that the psychological problems will lessen and students will mainly enjoy this competitions.

How can we achieve it? Let us start with the recognition that the classical mathematical Olympiad in the form of an exam is a game! This fact is not accepted even by many specialists, but that is a game! As a result of this recognition, we make the following conclusion: to make the mathematical competitions more attractive for a mass of students, we have to strengthen their game component.

Notes about Games in the Education Process

Games play an important role in child's development: through games, children obtain information on the world at large. In kindergarten and, to some extent, in elementary school, games are used to teach languages and science. However, teaching methods that use games have disappeared from use in high schools and in institutions of higher education, despite the fact that even at these ages, games can help

learners learn rapidly and with ease.

Our goal is to construct models, which could be used to involve school children and students in educational games. In all games, learners solve mathematical problems in their free time. This is clear to the teacher, but the activities are presented to students as a game.

The Blitz Mathematical Olympiad

One of the advantages of team competition is that nobody takes a full responsibility for the team's loss. This essentially solves negative psychological problems, which appeared in the classical individual competition.

The scenario of Blitz Mathematical Olympiad can be described as follows. All teams receive the same problem and have to come up with a solution. In order to score points, teams must submit the solution to the problem within the allotted time (10-15 minutes). The solutions are immediately checked by the panel of judges, who immediately announce the results to the participants and to the audience. All of this is repeated with the next problems. Dynamics of Blitz Mathematical Olympiad is one of the main principles. We inform participants about their results after solving each problem, rather than after solving all problems, which allows everyone to track the teams' score in real time.

The principle of scoring can be, for example, the following. The number of points for solved problem is inversely proportional to the number of teams who solved this problem correct. Consider, for example, the case of 4 teams. Suppose every problem is worth 12 points. In the situation that all of the teams solved correct, each team gets 3 points. If three teams solved correct, then each of them gets 4 points and the fourth team get 0 points. If two teams solved correct, each of them gets 6 points, and the other two teams - 0 points. In the case of only one right solution, this team gets 12 points and the other three - 0 points. It is important that the order of problems is from simple to more complicated problems. This usually allows us to hold non evidence of the final result till the last problems.

Another option is to propose an alternative scoring method. Every team starts with the same number of participants, for example, five. If a team solved a problem correct, one of its members can go out from the game. If after solving several problems correctly, a team has "disappeared" (all members went out of game), it means that this team won Blitz Mathematical Olympiad.

Modern Internet technologies allow us to organize Blitz Mathematical Olympiads when the teams stay at their Universities in different cities and even different countries.

Our Experience

Blitz Mathematical Olympiad was first organized in Perm Polytechnic Institute, Perm, Soviet Union in the 1980s. Blitz Mathematical Olympiads were regularly organized for high school students in the Department of Youth Activities in the Technion, Haifa, Israel in 1993-1999. In November 2008 the First International Blitz Mathematical Olympiads for University students from Russia, Romania and Israel was successfully organized in Ariel University Center, Israel.

As an example, how the problems of Blitz Mathematical Olympiad can be chosen, we present the problems from one of our competitions.

Problems of Blitz Mathematical Olympiad on November 18, 2008

1. Two friends have not seen each other for many years. When they met, they talked about their family status. "I am father of 3 children"- said one friend.

"How old are they?" – asked the other friend.

"The product of their ages equals 36; the sum of their ages equals the number of the bus that has just passed by!"

The other friend looked at the bus number and said: "there is not enough data to find out their ages!"

"My eldest son is blonde," – said the first friend.

"Then there is no problem," - said the second one.

"They are ?, ?, ? years old."

What was his answer?

2. Let $\{a_1, a_2, a_3, \dots, a_{100}\}$ be an geometric sequence

(a) Is it possible that all elements except a_{100} are integers?

(b) Is it possible that all elements except a_{50} are integers?

3. Post Airplane

Every day at the same time a post airplane arrives at the airport. At the same time a car from the central post office arrives to the airport to get the mail. One time, the airplane came early and its luggage was put on a horse carriage, which was on its way to the central post office. Half an hour after leaving the airport, the carriage came across with the post car, which left the post office as usual, and was on its way to the airport. Somebody put the mail from the carriage on the car, which drove back to the post office and arrived there 20 minutes earlier than usual. How early did the airplane arrive at the airport (don't take into account the time of unpacking the mail and putting the luggage on the car)?

4. Find all the integer solutions of the following equation:

$$y^5 = x^2 + x$$

5. Compute the determinant:

$$\begin{vmatrix} 1 & 2 & 3 & 4 & 5 \\ 2 & 3 & 4 & 5 & 6 \\ 3 & 4 & 5 & 6 & 0 \\ 4 & 5 & 6 & 0 & 0 \\ 5 & 6 & 0 & 0 & 0 \end{vmatrix}$$

6. A sequence is given by two first elements $a_0 = a_1 = 1$,

$$\text{and } a_{n+1} = \frac{a_n a_{n-1}}{\sqrt{a_n^2 + a_{n-1}^2}} \text{ for } n > 1.$$

Compute $\lim_{n \rightarrow \infty} \frac{a_n}{a_{n+1}}$

7. Is it true that for any polynomial $p(x)$ with real coefficients there is a polynomial $q(x)$ with real coefficients such that $q(x^3)$ is divisible by $p(x)$?

8. A chessboard is given, the length of a side of each small square is 1.

Compute the sum of areas of all rectangles, whose 4 sides go along the lines of the chessboard.

Note that the majority of these problems are based on the problems written by Prof. Alexei Kannel-Belov and Lev Radzivilovsky, coaches of the Israeli student team on mathematics, while several other problems were taken from the known Russian mathematical journal "Kvant" and from the book "Puzzle-head" by Ben-Zion Erez.