

Basic knowledge and Basic Ability: A Model in Mathematics Teaching in China

CHENG Chun Chor-Litwin

The Hong Kong Institute of Education, Hong Kong cccheng@ied.edu.hk

Abstract

This paper aims to present a model of teaching and learning mathematics in China. The model is “Two Basic”, basic knowledge and basic ability. Also, the paper will analyze some of the background of the model and why it is efficient in mathematics education. The model is described by a framework of “slab” and based on a model of learning cycle, allow students to work with mathematical thinking. Though the model looks of demonstration and practice looks very traditional, the explanation behind allows us to understand why Chinese students achieved well in many international studies in mathematics. The innovation of the model is the teacher intervention during the learning process. Such interventions include repeated practice, and working on group of selected related questions so that abstraction of the learning process is possible and student can link up mathematical expression and process. Examples used in class are included and the model can be applied in teaching advanced mathematics, which is usually not the case in some many other existing theories or framework.

Introduction

China started reviewing her mathematics curriculum in the early eighties. Chinese learn mathematics in a very traditional way, mostly by imitation and practice. Many thought that such delivery of mathematics education is not good for the students. However, it is also a fact that many Chinese students achieve well in mathematics standard and they did not lose their interest in mathematics. Most of the time, discussion of learning theory in China focus on western scholar such as Dienes, Freudenthal etc. Very few of the Eastern scholars and theory are mentioned.

There are a lot of developments in the near 100 years in learning in the West. For example, the nonassociation theory of thinking, explanation of learning through behaviourism and Gestalt, and the development of cognitive science by Piaget. The following is a list of theories of learning mathematics in the West that is related to the learning of mathematics.

Dienes and multiple embodiments

Dienes maintained that structured teaching aids can help students to establish their concepts. And play is important for learning. Dienes proposed that learning of mathematics can be more efficient with multiple embodiments. The design of instruction is described by the grids in the box representing the increase in depth in mathematics and the increase in abstraction in perceptions.

RME in Holland, Van Hiele and Freudenthal

The development of cognitive science resulted in RME (Realistic Mathematics Education) brought by Freudenthal in his process of mathematization in mathematics education. Van Hiele proposed that the stages learning go through “visual, analysis (part and whole), abstraction,

informal deduction, formal deduction”. The characteristic of RME is related to Van Hiele’s levels of learning mathematics.

Problem Solving by Polya

Polya proposed three principles of mathematics learning, which is active learning, best motivation, and stages of progression (exploratory phase, formalizing phase, assimilation phase).

APOS by Dubinsky

Dubinsky proposed that learning of mathematics is through APOS (Action□Process□Object□Schema)

Richard Skemp, Schema and principles in learning concepts

Skemp introduced his idea of schema, instructional knowledge and relational knowledge. Skemp consider that concepts are formed from abstraction of context, which came through from abstraction of operation. Mathematical knowledge is a special kind of abstraction.

David Tall – Procepts and proceptual divide

Tall proposed that students are troubled by procedure and concept during their learning. Hence they could not realize the real meaning of the concept. Tall proposed a term of Pro-cept, which is a combination of procedure and concept. Procepts compressed Process, and if students can cross the perceptual divide, then they are able to compress their knowledge.

Origin of the Two Basic models in learning mathematics

In China, learning is related to “comparison of object and emerge into knowledge” has a long history dating more than 1000 years in China. Teachers have a mentor role, demonstrate and intervene. Students imitate and internalized. Most of the knowledge we acquired are not from direct personal experience, but indirect experience, such as schooling. And schooling provides experience in learning mathematics by discovery approach and re-invention.

Two Basic Models in Learning Mathematics (Zhang Dianzhou)

Two basic means “Basic Knowledge” and “Basic Skills”. According to Zhang, high-level knowledge and ability are intervened. Skills can be developed into knowledge and knowledge can induce skills. This also echoes the old Chinese saying, knowing and practicing. In fact, skills and knowledge overlap at some instances.

Metaphor of Knowledge formation in Two Basic, building of slabs

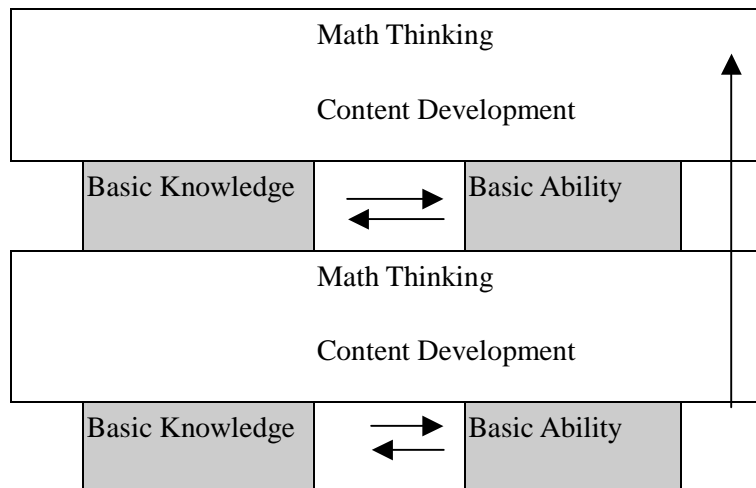
Zhang has proposed a metaphor of knowledge formation in the "two basic model", building of slabs. Basic knowledge is similar to reinforcement in the slabs. There are two kinds of knowledge, direct knowledge through exploration and investigation, and these reinforcements are thick. The second kind of knowledge is indirect knowledge. They came from imitation, there reinforcement are many but thin. Basic ability is the concrete in the slab, with the suitability reinforcement, the slab formed is strong.

Two basic are the slabs, these slabs connected together by ability. The successful of solving the problem depends on whether such slabs are connected. Connection of slabs means connection

of knowledge.

The quality of foundation depends on number of reinforcement and also the quality of the concrete, and also how the slabs are joined together. As knowledge structure is not conceived in the same format for everyone, structure of knowledge is differently arrayed in every learner.

Slab Model of the “Two Basic”



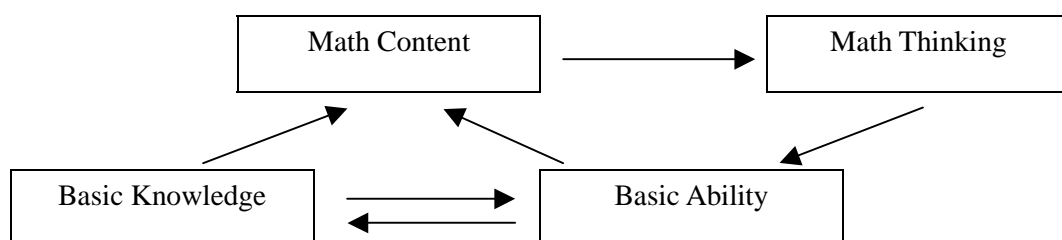
The three steps of Two Basic Model-(1) Imitation and Representation, (2) Intervention, and (3) Abstraction and Internalization

The first step is imitation, with teacher’s guidance and demonstration. Demonstration is the premiere of imitation. Imitation means students need to observe and internalize. The second step is the intervention by teachers. This includes criticism, correction of concepts, and using more strong examples for the concepts, and the summary of knowledge learned in a trunk and practice. These will raise the level of understanding of the students. The third step is abstraction process by students, which is a kind of internalization and self monitoring. By internalization, students can connect different knowledge, and deduce new knowledge.

Step 1		Step 2		Step 3
Imitation		Teacher’s Intervention		Internalization and Abstraction
Observation and representation Internalization (level 1)	→	Correction and reinforcement of concept	→	Connect to different domain of mathematics Deduce mathematics

And we can present the model of the learning cycles as following.

The Model of Learning cycles in the “Two Basic”



Imitation and intervention before construction

The Chinese theory attained that construction is the process of imitation and intervention. It is achieved through two levels. The first level is imitation of what is taught by teachers, and the second level is construction of knowledge through internal abstraction. And practice has an important role. How to avoid having practice becomes mechanical repetition? The two basic provide the following principles:

- (1) memory leads to recognition and become intuition,
- (2) good speed of operation provide grounds for efficient thinking,
- (3) using deduction and reasoning to sustain precise logical thinking,
- (4) rising of standard through variation of problems and learning process.

Zhang maintained that teacher's direct demonstration has a strong role in establishing learning objective, and provide guidance during learning. Learning of a language is an obvious example. Such imitation and internalization can also found in the domain of mathematics learning. And construction is based on intuitive of concepts so that formal concept is built. He attained that western theories stressed the importance of construction, but have skipped the important role of imitation. Knowledge that can be constructed in a short while is only very surface knowledge.

How classroom teaching process leads to mathematical thinking

There are two levels of teaching. The first one is using daily lives context, which aims to arouse the interest of students. The second level is the learning process through abstraction. Teachers teach mathematics based on the process of "correspondence, induction and deduction". Correspondence means to map the concept of mathematics problem to another problem; it also involves correspondence of expression, and structure. For example, learning of fraction division correspondence to integers division. The induction process involves the process pattern of calculations and then generalizes the pattern. And by deduction, students deduce related results of the generalized pattern. For example, $4 \times 3 = 12$, $4 \times 4 = 16$, $4 \times 3.5 = ?$ Through deduction, the answer is half of $(12 + 16) = 14$. The process can continue to 4×3.25 , which is half of $(4 \times 3 + 4 \times 3.5)$.

Example: Integers divided by fractions

Question:

A walk 1 km in $\frac{1}{4}$ hours. How much distance can he covered in an hours?

The answer is $1 \div \frac{1}{4}$. The answer is not demonstrated by teachers. The answer is obtained through interaction between teachers and students. The process is by using analogy and related questions. Using intuition and multiple, the answer is 4 km.

