Linking Geometry, Algebra and Calculus with GeoGebra

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Abstract

GeoGebra is a free, open-source, and multi-platform software that combines dynamic geometry, algebra and calculus in one easy-to-use package. Students from middle-school to university can use it in classrooms and at home. In this workshop, we will introduce the features of GeoGebra with a special focus on not very common applications of a dynamic geometry program. We will inform about plans for developing training and research networks connected to GeoGebra.
We can expect that at the time of the conference a spreadsheet will be integrated into GeoGebra which offers new ways teaching mathematics using the interplay between the features of a spreadsheet and the objects of dynamic geometry.

Workshop Examples

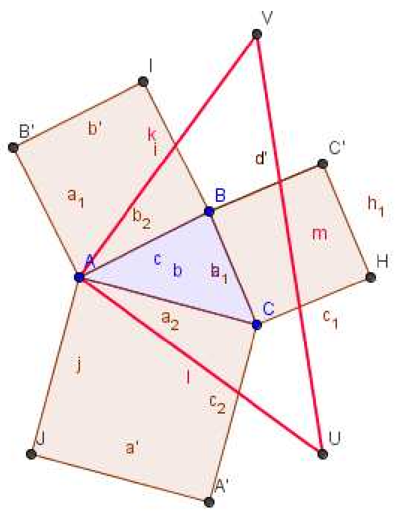
We will adjust the examples to a possible previous knowledge of the participants and make a selection of the following problems according to the wishes of the audience.

1 Another Property of a Triangle

Given is an arbitrary triangle ABC. Construct squares over all sides of ABC. Take two vertices of the triangle – say B and C – and complete two adjacent sides of the respective squares to parallelograms. The resulting fourth vertices U and V of the parallelograms are connected with the remaining vertex of the triangle – A.

Try to use macros for your GeoGebra construction!

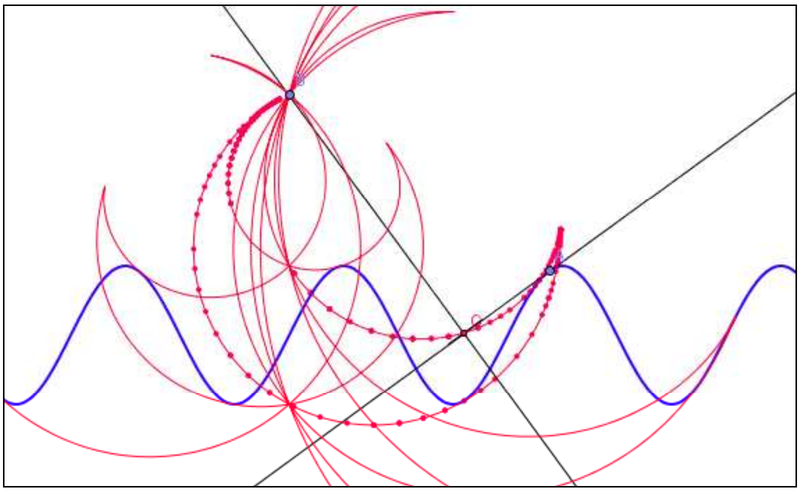
Do you have any conjecture about the properties of the generated triangle AUV?
Can you prove your conjecture?
What happens if the squares are erected in the other directions?
Can you find a formula for the area of the triangle?
We will show a generalized result given by Geometry Expressions.



Credit goes to Peter Lücke-Rosendal. He found this problem in an US-journal.

2 Some Calculus – Finding Pedal Curves and Relatives

1. Given is a curve and a fixed point B (the pole).
2. Draw a tangent at any point A on the curve.
3. Mark a point Q on this tangent so that BQ and AQ are perpendicular
4. Find the locus of all points Q when A is tracing the curve. This locus is the pedal curve of the given curve with respect to the pole.

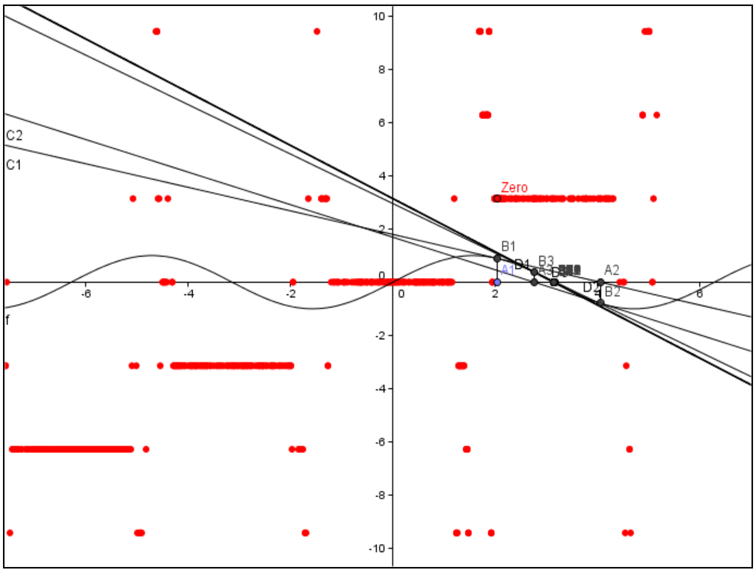


The graph shows a pedal curve of a sine wave.

Further investigations: Find the Negative Pedal Curve, find the Contrapedal.
Find the equations of these curves using a CAS and compare with the loci produced supported by GeoGebra.

3 Using the Spreadsheet for the Newton-Raphson-Algorithm

We can assume that everybody is familiar with the Newton-Raphson Algorithm for finding roots of equations numerically. We will show in a dynamic way the dependency of the root on the chosen initial value.

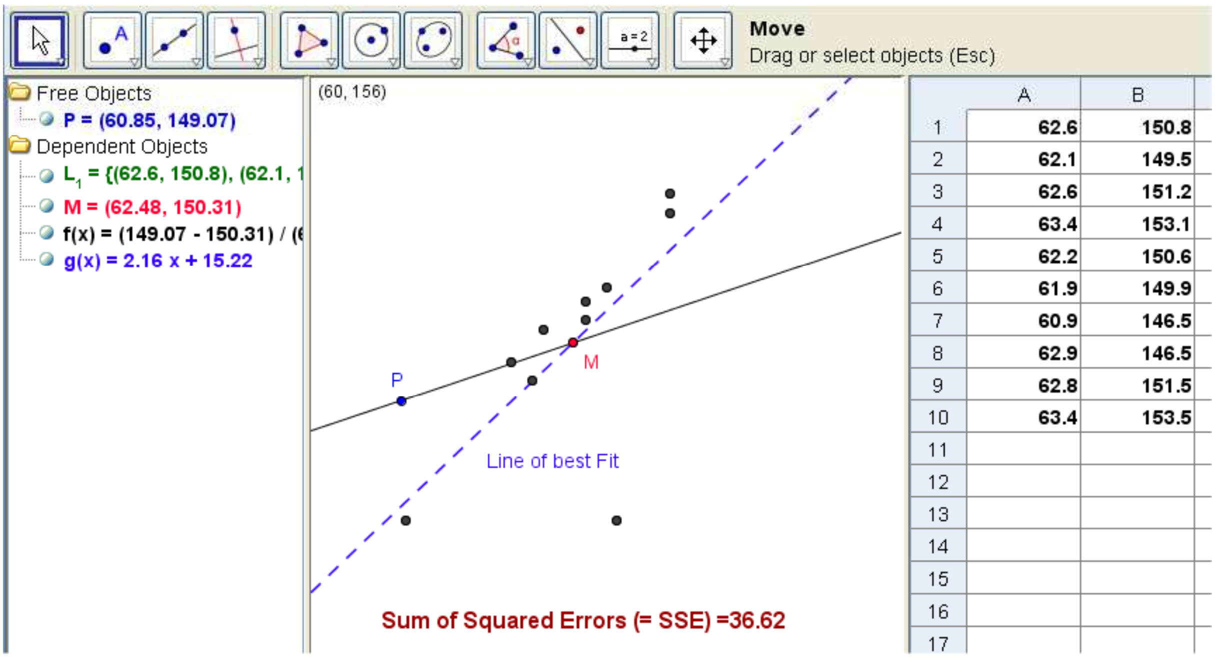


The red points - ini_val,zero(ini_val) - represent the result of the algorithm as a function value of the chosen initial value. A chaotic pattern appears. Zooming in will give interesting insights.

4 Some Statistics with GeoGebra

This is an example which can be found in an Austrian textbook ("Mathe mit Gewinn" = "Maths with Profit"). Given are the height, the length of the stride and the shoe size of 10 students. The problem is to find a relation between the height and the stride (the height and the shoe size).

Name	Größe cm)	Schrittw.(cm)	US-Schuhgröße
Josephine	150.8	62.6	4
Carl	149.5	62.1	5.5
Stanley	151.2	62.6	6.5
Terence	153.1	63.4	7.5
Larry	150.6	62.2	7.5
Walter	149.9	61.9	5
Patricia	146.5	60.9	4.5
Eleonor	146.5	62.9	6
George	151.5	62.8	8.5
William	153.5	63.4	6.5



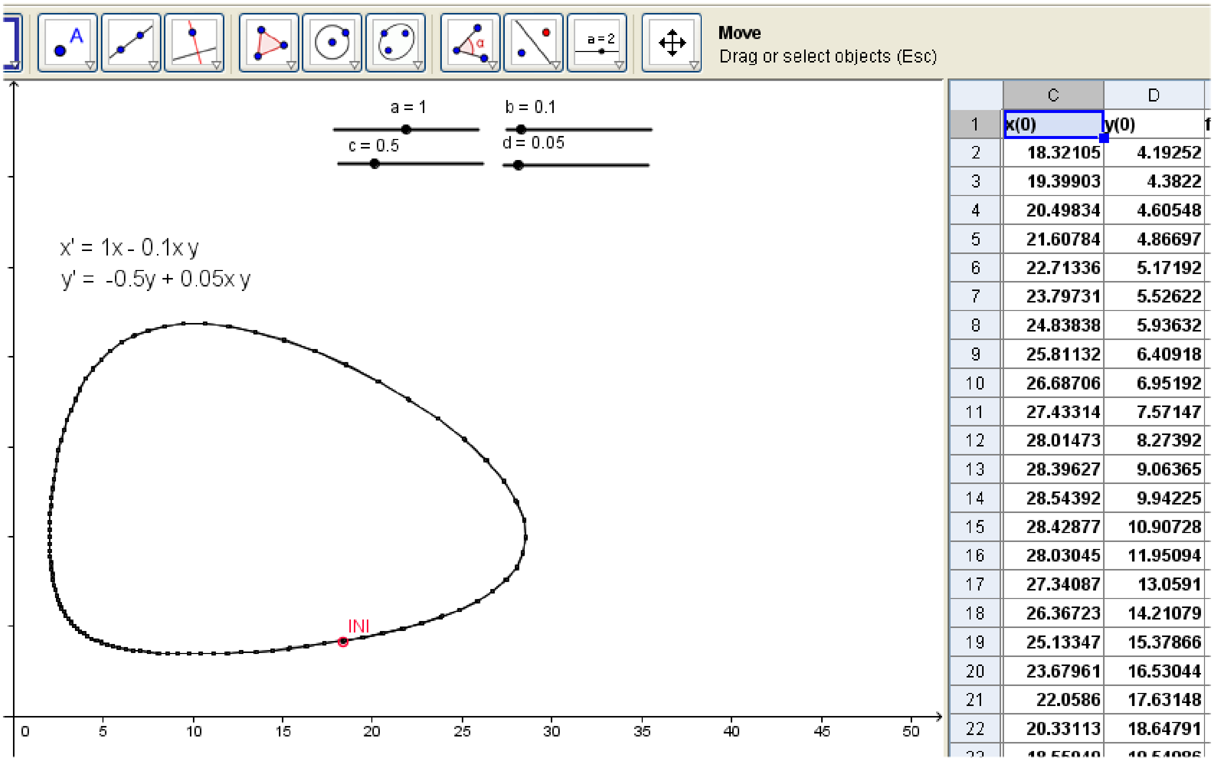
The students enter the data and represent them in form of a scatter diagram. They are instructed to find the "mean point" and any line passing this point. Then they shall find the position of the line giving the minimum of the sum of the squared errors.

Finally we add the linear regression line and compare the results.

Do the same for shoe size vs height and discuss the outcome.

5 Prey-Predator with the Improved Euler Method and Sliders

This is one of the classic problems in dynamic systems. We will use the spreadsheet together with sliders to make the model as flexible as possible. Students might be inspired to realize better numeric routines like Runge-Kutta-method (and worse routines like Euler-method).



You can see the mathematical model (in form of a dynamic text). The sliders control the parameters. The initial point can be moved by the mouse. The phase diagram changes accordingly. We can add the plots of both populations vs time.