Origami-Mathematics Lessons: Researching its Impact and Influence on Mathematical Knowledge and Spatial Ability of Students

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Abstract

"Origami-mathematics lessons" (Boakes, 2006) blend the ancient art of paper folding with the teaching of mathematics. Though a plethora of publications can be easily found advocating the benefits of Origami in the teaching of mathematics, little research exist to quantify the impact Origami has on the learning and building of mathematical skills. The research presented in this paper targets this common claim focusing on how Origami-mathematics lessons taught over an extended period of time impact students' knowledge of geometry and their spatial visualization abilities. The paper begins with a brief overview of Origami as it relates to teaching mathematics followed by a summary of research done with two age groups: middle school children and college students. Gathered data in these two studies suggest that Origami-mathematics lessons are as beneficial as traditional instructional methods in teaching mathematics.

Introduction

Mathematics stands as an essential part of a child's education. Beyond the obvious every day applications, mathematics is seen as a needed element in developing student readiness for the workforce demands of the 21st century (National Council of Teachers of Mathematics, 2000). Though this is widely recognized and accepted the National Council of Teachers of Mathematics (NCTM), the largest mathematics association in the United States, calls for continued improvement in methods of teaching mathematics that engage, excite, and develop mathematical thinkers (2000). A recent report put out by the National Mathematics Advisory Panel (2008), commissioned by United States Department of Education (USDOE), concurs citing students lagging scores on international assessments like the Trends in Mathematics and Science Study (TIMSS) and Program for International Student Assessment (PISA). When compared to our international economic competitors, US students rank 8th and 9th out of 12 countries reviewed. Of the topic areas reported, the study of geometry was identified as a significant area of weakness on international assessments.

The word geometry broke down into parts stands for "geo"-earth and "metry"-measure. Though this topic has an obvious tie to our natural world and is a core element of mathematics, it stands as one that US students struggle to grasp. NCTM's standards describe geometry as the study and analysis of shapes and structures (2000). Part of this study includes the development of spatial visualization defined as "[the] building and manipulating [of] mental representations of two- and three-dimensional objects and perceiving an object from different perspectives" (NCTM, 2000, p.41). It is this skill that children use to describe, interpret, and understand their natural surroundings. To develop these skills, NCTM calls for teachers to actively engage students through a variety of hands-on, engaging tasks. Of these experiences, once such tactic noted is "paper folding".

The use of paper folding as a way to engage students in mathematical thought is far from a new concept. Publications in the US have featured its benefits since the 1960s with the printing of *Geometry Exercises in Paper Folding* (Sundara Rao, 1966). As of 2007, 27 books relate Origami to teaching mathematics (Tubis, 2004). NCTM alone has published 8 articles in their national teaching magazines featuring the benefits of Origami as an instructional tool. Internationally, paper folding in the classroom dates back to the 1800s when the founder of kindergarten, Froebel, included the art in his curriculum as a way of promoting children's mental growth and grasp of basic geometry. In all cases, Origami is seen as a powerful tool to teach mathematical concepts, particularly in geometry (Boakes, 2006). Writers speak of a variety of benefits including familiarization with geometric figures & principles (Pearl, 2008), developing spatial sense (Robichauz & Rodrigue, 2003), and engaging children in the discourse of mathematics (Cipoletti & Wilson, 2004).

Beyond the tie of Origami to mathematics, the art of paperfolding also has links to learning theories (Boakes, 2009). For instance, within Piaget's work on cognitive development Piaget discusses the development of logical-mathematical intelligence. Emphasized is the need for children to construct and develop their own meanings of mathematics through physical manipulation and play. Learning modalities and preferences also relate well. Modalities deal with how we prefer to learn information when first presented. There are three major categories within modalities that are universally accepted including auditory, kinesthetic, and visual-spatial. Auditory learners make sense of knowledge through what they hear. Kinesthetic learners, as the name suggestions, use touch and movement as a way to grasp presented concepts. The final modality, visual-spatial, relates to those that need visual stimulus and images to help understand material presented. By addressing all modalities in instruction, teachers are more likely to have success in reaching all children.

Learning preferences (also known as learning styles), slightly different from modalities, deal with the way in which a student processes information once it is presented. Known best in this field is Martin Gardner. His theory speaks of a set of multiple intelligences that all individuals possess. These intelligences include linguistic, logical-mathematical, spatial, bodily kinesthetic, musical, naturalistic, interpersonal, intrapersonal, and existential. Students taught in ways that allow them to explore these various intelligences tend to be more motivated, engaged, and retain more of what they are taught. Consider how Origami relates to these theories. The practice engages children physically requiring listening and visual stimulation. The act of folding involves spatial skills and geometric shapes. It is for this reason that Origami seems to have captured the interest of those that teach mathematics and has been accepted as a beneficial practice in the classroom.

One nagging issue remains among all that has been presented thus far. It's clear that there is room for growth in the area of geometry instruction. The national mathematics standards of NCTM support the use of such hands-on, physical tactics for learning geometry. Origami is an accepted method for teaching mathematics and relates easily to learning theories. However, how can one be sure that Origami truly is a beneficial experience in the mathematics classroom? To answer this question the next logical thing to do is to seek out research that quantifies the impact Origami has when instructed in a mathematic classroom. Interestingly, there is very limited research to substantiate what we think is true about Origami and mathematics (Boakes, 2006). It is this fact that prompted the research presented in this paper on the effect Origami instruction has on an individual's mathematical knowledge and geometry skills.

Anatomy of an Origami Mathematics Lesson

In both sets of research conducted, the term "Origami-Mathematics Lesson" was coined for the treatment method in the studies. This term "refers to a mathematical lesson taught using an origami activity linking students' mathematics knowledge and skill during the folding process and with the resultant Origami figure" (Boakes, 2006). In a normal folding session, an instructor of Origami would verbally and visually show each folding step. Origami-mathematics lessons simply blend mathematical terminology and discussions within this process. An article found in the online journal Mathitudes details this process and provides an example of what an Origami-mathematics lesson might include (Boakes, 2008).

Research with Middle School Students

The first research study was conducted in a suburban middle school in southern New Jersey. The purpose of the study was to "compare the spatial visualization abilities and mathematical achievement of seventh-grade students taught by two different methods of instruction" (Boakes, 2006, p.82). To do so, a basic quasi-experimental design was used. A control group of 31 students received traditional instruction over the course of a month long geometry unit. Meanwhile, a treatment group of 25 students received traditional instruction along with the infusion of a collection of 12 Origami-mathematics lessons. Both sets of students were taught by the same classroom teacher. The researcher served as the instructor for all Origami-mathematics lessons. During sessions where Origami-mathematics lessons were taught, about 20 minutes was taken from the normal 80 minute class held daily.

The groups were pre- and post-tested using an excerpt from a national mathematics assessment and a set of spatial tests. The mathematics achievement test used contained 27 multiple choice questions within the geometry/spatial skill strand of the National Assessment of Educational Progress (NAEP) assessments better known as the "Nation's Report Card" (National Center for Educational Statistics, 2004). Three subtests measured students' spatial abilities through a card rotation, paper folding, and surface development test taken from the Kit of Factor-Referenced Cognitive Tests (Ekstrom, French, Harman, & Derman, 1976). The card rotation test measured students' ability to mentally manipulate and analyze 2-D figures. The paper folding test requires the student to imagine folding and unfolding a square sheet of paper. The final of the three spatial tests, the surface development test, has readers try to match parts of a 3-D geometric figure with its 2-D net.

Data on the quantitative assessments was analyzed using an analysis of covariance (ANCOVA). The ANCOVA allowed the researcher to control for initial differences in spatial skills and mathematics achievement levels. A summary of results by group are presented in Table 1. The analysis completed using statistics software was a 2x2 between groups ANCOVA to determine the differences in mean scores among groups and gender. Looking at variables individually, no statistically significant differences were found among any of the 4 tests. However, a significant interaction effect [$F(1,51)=3.59$, $p=0.64$] was found on the Card Rotation Test between gender and group where males in the experimental group and females in the control group earned higher gains in scores than their counterparts in the same groups.

Table 1
Descriptive statistics for all instruments with middle school age students

Instrument	**Group**	**N**	**Pre-test Mean**	**SD**	**Post-test Mean**	**SD**
Card Rotation Test	Control	31	52.29	14.85	60.00	13.52
	Treatment	25	56.56	17.46	60.04	16.22
Paper Folding Test	Control	31	3.61	1.56	4.32	1.89
	Treatment	25	4.04	2.34	5.00	2.14
Surface Development Test	Control	31	10.16	7.13	13.84	8.04
	Treatment	25	10.00	6.98	14.28	8.30
Mathematical Achievement Test	Control	31	14.97	3.86	15.68	3.96
	Treatment	25	14.00	4.38	16.60	3.91

Without delving into gender differences and instead looking more generally at the overall results, data were not strong enough to warrant statistical significance. While this is true, if one examines Table 1 it can be said that both groups improved their mean average score on all tests. Further, the treatment groups had a similar or higher mean score than that of the control group. There are many reasons for this occurrence, only one of which may be the infusion of Origami-mathematics lessons. It is fair to conclude though that this instruction method was as beneficial as traditional instruction in the mathematics classroom and stands as an acceptable tool for improving children's spatial skills and geometry knowledge.

Research with College Level Students

A second study was conducted during the spring terms of 2008 and 2009 at the Richard Stockton College of NJ to further investigate the impact of Origami-mathematics lessons on students' abilities. In this case, a college course called The *Art and Math of Origami* was utilized. The course is similar in purpose and structure to that of Alan Russell's (2007) Origami course, *Mathematical Origami*, discussed at the Ninth International Conference of the Mathematics Education in the 21st Century Project. Students studied the art of paper folding while also learning about the history, culture, and mathematics that relate to it. The researcher, who served as the instructor of this course, infused Origami-mathematics lessons into the 27 two-hour sessions. A total of about 10 sessions were dedicated specifically to the tie between geometry and the folding process.

Students in this study ranged in age from 20 to 45 years old and were mainly junior & senior level based on the number of credits earned. Data gathered was the culmination of two groups of students, 24 during spring 2008 and 23 during spring 2009. Due to the difference in age of this group versus the previous study containing seventh graders, the assessments used to determine change in mathematical abilities was limited to the three spatial tests. (These tests are rated as appropriate for both age ranges.)

Pre- and post-test data were gathered on all three spatial tests. Since all students experienced the Origami-mathematics lessons in this study the data analysis method used was a paired-sample t-test. Results are shown below in Table 2. In all cases there was a significant increase

Table 2

Paired sample t-test statistics for the college age groups

	Pre-test	SD	Post-test	SD	Mean	SD	*t*	df	Sig.
Card Rotation Test	87.23	51.51	90.98	48.64	-3.75	12.21	-2.10	46	.04*
Paper Folding Test	12.13	3.32	13.43	3.80	-1.30	2.23	-4.00	46	.00**
Surface Development Test	39.60	13.09	45.87	11.13	-6.28	7.23	-5.95	46	.00**

*p<.05, **p<.005

seen in students' mean scores on all three spatial tests. Eta-squared values calculated indicate moderate (.09 for card rotation) to large effect sizes (.26 for paper folding & .44 for surface development). These results seem to support the conclusion that Origami does indeed have an impact on students' spatial skills. However, because this is a group of diverse individuals with varied academic backgrounds it is difficult to say with certainty that Origami was the sole cause of such change. Based on this being done over a short period of time however it is likely to be a contributing factor.

Discussion & Conclusion

The attempt in both studies was to examine the claim that Origami is an effective teaching tool capable of strengthening students' mathematical and spatial abilities. This paper provides an abbreviated review of each study conducted on very different groups of individuals. In the case of the middle school students, increases in both knowledge and spatial skills were seen with treatment students doing as well or better than their control student counterparts. While this is true, these differences were not pronounced enough to find statistical significance. There are a variety of reasons this may have occurred including the fact that this study was run over a very short period of time with a limited number of students. A fact worth noting though is that while 20 minutes of a daily lesson was taken away in order to fit in the

Origami-mathematics lesson, treatments students performed as well overall on 3 of the 4 tests used including the mathematics achievement test. For the group of college students, results were all significant. A large increase in the mean score received was found for all three spatial skills tested. Though again it cannot be said for sure what caused this increase with great certainty, it stands to reason that the 4 hours of Origami instruction received per week (plus work outside of course hours) may very well have contributed.

Origami, mathematics, and teaching have had a long standing relationship. In this paper, you've learned why this is the case. It was the purpose of the research conducted to shed light on whether this coupling of ideas is beneficial for students. Based on what's been presented, there is definitely potential shown for Origami-mathematics lessons. While there is still a great need for research in this field to be certain of this claim, these results should warrant inclusion of such practices in the mathematics classroom.

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