Comparing Mathematics Education Traditions in Four European Countries: The Case of Teaching Percentages in the Primary School

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Background and objective

• Part of the METE-project
• Small-scale videobased comparative study
• Four participating countries: Flemish Belgium, England, Hungary, and Spain
• Topics: percentages and polygons (upper primary school, age 10-12), and polygons and linear equations (lower secondary school, age 12-14)
• To identify distinctive features of the math education traditions and to understand them within their specific context
1. Perspective on the teaching of %

1.1 Objectives

- **Computational goals**
  - Master one (or more) procedure(s) to compute %
  - Procedural knowledge

- **Conceptual goals**
  - Deep understanding of the concept %
  - Conceptual knowledge

- **Applicational goals**
  - Apply % in all kind of (meaningful) situations → adaptive expertise
  - Interconnection of procedural and conceptual knowledge
1.2 Conceptual aspects

- % refers to something
  e.g., You need 50% correct answers to succeed. Loes solved 14 tasks wrongly. Can we congratulate Loes or not?

- % describes a fixed situation
  e.g., Black currant jam, which contains 60% of fruit, is sold in large (450g) and small (225g) pots. Someone forgot to put the percentage of fruit on the small pot. How many grams of fruit does each pot contain?

- Adding/subtracting %: non-linear
  e.g., \((\text{whole}_1 + 20\%) + 30\% \neq \text{whole}_1 + 50\%\)

- % describe two types of situations
  - Part of a whole
    e.g., Bread consists of 73% flour, 25% water, 1% yeast, and 1% butter
  - Whole +/- a part
    e.g., 100 visiting cards including V.A.T. cost €10. What’s the price excluding V.A.T.?
1.3 Didactic tools

- Everyday situations and students’ informal knowledge
e.g., 50% = \( \frac{1}{2} \)

- Relationship with other mathematical entities
e.g., fractions, ratios, decimal numbers

- Simultaneous use of different models
e.g., bar model, elastic percent meter, slide-slip
2. Aims

- Understanding mathematical practices within their specific educational context
- Does not aim at evaluation and generalization

3. Methodology

- Videotapes of a sequence of 4 or 5 % lessons
- Upper primary school (age range of 10-12)
- 4 European countries: England, Flanders, Hungary and Spain
- 2 instruments
  - Lesson synthesis sheet
  - Lesson coding scheme
- Framed within the perspective on teaching
3.1 Methodology

- Videotapes of a sequence of 4 or 5 percentage lessons in each participating country
- Teachers were representative of good but not exceptional teaching practices in their country
- Collection of information about the local context
- 2 analysing schemes: a lesson coding scheme and a lesson synthesis sheet: based on live observations spread over one week of a series of lessons by members of the four country teams in each country (including videotaping)
- Results were subjected to a Mann-Whitney U-test: to determine the influence of the approach
3.2 Methodology: Lesson coding scheme

- **4 basic categories**
- **Mathematical focus:** underlying objectives of teacher’s actions/decisions
  - **Subcategories:** conceptual, derivational, structural, procedural, efficiency, problem solving, reasoning
- **Mathematical context:** conception of mathematics underlying the tasks
  - **Subcategories:** real world fabricated data, not real world fabricated data, real world genuine data, not real world genuine data
- **Didactics:** teacher’s didactic strategies
  - **Subcategories:** activating prior knowledge, exercising prior knowledge, explaining, sharing, exploring, coaching, assessing/evaluating, motivating, questioning, differentiation
- **Materials:** teacher’s and students’ use of concrete materials
  - **Subcategories:** answer book, blackboard, computer, calculator, coloured writing materials, display materials, game, pupil whiteboards, manipulatives, overhead projector, practical equipment, real world materials, worksheet, textbook
- **Coded for every episode of the lesson**
  1 if present and 0 if absent
3.3 Methodology: Lesson synthesis sheet

- Based on the work of Reusser (www.didac.unizh.ch)
- Brief details about the lesson involved
- Photograph of the classroom
- Two-dimensional timeline:
  **Pedagogic activities**: subcategories: theory/conceptual development, working on problems/tasks, reporting solutions to problems/tasks, introducing a problem/activity, homework-related activities, task-related management, non-task-related management
  **Social activities**: subcategories: whole class, group, paired, and individual activities
- Narrative summary, linking a qualitative description of the lesson to the two timelines and to the categories of the lesson coding scheme
M.E.T.E. summary sheet for video recorded lessons

Country: Belgium
School: Paridaens; Leuven
Class: grade 6; middle ability
Teacher: Patrick Van Welde
Date: 05/11/2003
Topic and focus of lesson: The periphery of a circle (3)

Lesson Duration: 56'03"

Pedagogic activity
- Red = Theory or conceptual development
- Yellow = Working on problems or tasks
- Green = Reporting solutions to problems or tasks
- Blue = Introducing a problem or activity
- Orange = Homework-related activities
- Purple = Task-related management

Social activity:
- Black = Whole class activity
- Light grey = Individual activity
- Dark grey = Paired activity
- White = Group activity
- Pink = Non task-related management
4. Results

4.1 Analyses of the lesson coding schemes
**Similarities:** A lot of time spent on: working on and reporting solutions to problems/tasks
Less time spent on: introducing a problem/activity, task-related and non task-related management

**Differences:** Great difference in theory/conceptual development, homework-related activities
**Similarities:** Whole class activities are dominant; working on individual activities have the second highest frequency

**Differences:** No paired and group activities in the Hungarian lessons.
4.2 Analyses of the lesson coding schemes

Similarities: strong conceptual and procedural focus

Differences: English lessons: significantly more conceptual (Z = -1.96; p = 0.05); significantly less efficiency (Z = -2.23; p = 0.03)

Hungarian lessons: significantly more derivational (Z = -2.35; p = 0.02), structural (Z = -3.33; p = 0.00), efficiency (Z = -3.00; p = 0.00), and reasoning (Z = -2.43; p = 0.02); significantly less procedural (Z = -2.221; p = 0.03)
**Similarities**: ± 2/3: not real world fabricated data; ± 1/5 real world fabricated data

**Differences**: No statistically significant differences between the different approaches to teach percentages
Similarities: 4 frequently used didactics: sharing, questioning, explaining, coaching

Differences: English lessons: significantly more activating prior knowledge ($Z = -2.01; p = 0.04$), exercising prior knowledge ($Z = -3.45; p = 0.00$), explaining ($Z = -2.53; p = 0.01$); significantly less motivating ($Z = -2.25; p = 0.02$) and questioning ($Z = -2.95; p = 0.00$)

Hungarian lessons: significantly less differentiation ($Z = -2.21; p = 0.03$)

Spanish lessons: significantly more motivating ($Z = -3.15; p = 0.00$)
4.3 Framing the 4 approaches within the current perspective on teaching %

- Objectives
  - Computational goals
    - Strong procedural focus
    - Difference in the kind of procedures that were taught
e.g., dividing the given amount by hundred to calculate 1% of that amount, and multiplying that result by the %;
OR percentage-web: all % were related to 10%
  - Conceptual goals
    - Strong conceptual focus
  - Applicational goals
    - Mainly in the Hungarian approach: wide variety of tasks, each of which needed an appropriate solution method
e.g., % and solution were given, and the students had to find the original amount
OR exercises that contained a combination of increase and decrease of certain %
Conceptual aspects

- Strong conceptual focus
- Main focus on “basic ideas” of %
  - e.g., % always expresses something out of 100; 100% = whole
- In all approaches: different tasks reflect 2 types of situations:
  - part of whole and whole +/- part
- Different aspects of % that lead to deep understanding:
  - hardly addressed
- Hungarian approach focused more on the different aspects that lead to a deep understanding of %
  - e.g., non-linear character
- Flemish approach: to a less extent
  - e.g., % describe a fixed situation
- **Didactic tools**
  - Everyday situations and students’ informal knowledge
    - When introducing the concept
e.g., sales
    - Only to a small extent: real world genuine data and real world materials
  - Relationship with other mathematical entities
    - Fractions in all approaches
    - Hungarian approach: decimals, degrees of a circle
e.g., How many degrees equal 1% of a circle?
  - Models
    - Manipulatives in all approaches
e.g., MAB-material, place-value cards
    - Models: ten by ten grid (F, H, S); pie chart model (E, H); arrow scheme (F, H)
5. Discussion

• **Scope:** small-scale videobased comparative study

• **Objectives:**
  not to evaluate or generalize the teaching of percentages in the participating countries rather to make an inventory of the variety in the different possibilities and traditions

• **Methodology:**
  development of instruments: time-consuming process
  use of multiple methods → qualitative data
  complemented quantitative data

• **Main results:**
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<th></th>
<th>SPAIN</th>
<th>HUNGARY</th>
<th>ENGLAND</th>
<th>FLANDERS</th>
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<td>Individual activity</td>
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