Comparing Mathematics Education Traditions in Four European Countries: The Case of Teaching Percentages in the Primary School

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Background and objective

- **Part of the METE-project**
- **Small-scale videobased comparative study**
- **Four participating countries: Flemish Belgium, England, Hungary, and Spain**
- **Topics: percentages and polygons (upper primary school, age 10-12), and polygons and linear equations (lower secondary school, age 12-14)**
- **To identify distinctive features of the math education traditions and to understand them within their specific context**

1. Perspective on the teaching of %

1.1 Objectives

- **Computational goals**
  - Master one (or more) procedure(s) to compute %
  - Procedural knowledge
- **Conceptual goals**
  - Deep understanding of the concept %
  - Conceptual knowledge
- **Applicational goals**
  - Apply % in all kind of (meaningful) situations → adaptive expertise
  - Interconnection of procedural and conceptual knowledge

1.2 Conceptual aspects

- **% refers to something**
  e.g., You need 50% correct answers to succeed. Loes solved 14 tasks wrongly. Can we congratulate Loes or not?
- **% describes a fixed situation**
  e.g., Black currant jam, which contains 60% of fruit, is sold in large (450g) and small (225g) pots. Someone forgot to put the percentage of fruit on the small pot. How many grams of fruit does each pot contain?
- **Adding/subtracting %: non-linear**
  e.g., $(\text{whole}_1 + 20\%) + 30\% \neq \text{whole}_1 + 50\%$
- **% describe two types of situations**
  - Part of a whole
    e.g., Bread consists of 73% flour, 25% water, 1% yeast, and 1% butter
  - Whole +/- a part
    e.g., 100 visiting cards including V.A.T. cost €10. What's the price excluding V.A.T.?

1.3 Didactic tools

- **Everyday situations and students' informal knowledge**
  e.g., 50% = ½

- **Relationship with other mathematical entities**
  e.g., fractions, ratios, decimal numbers

- **Simultaneous use of different models**
  e.g., bar model, elastic percent meter, slide-slip

2. Aims

- **Understanding mathematical practices within their specific educational context**
- **Does not aim at evaluation and generalization**

3. Methodology

- **Videotapes of a sequence of 4 or 5 % lessons**
- **Upper primary school (age range of 10-12)**
- **4 European countries: England, Flanders, Hungary and Spain**
- **2 instruments**
  - Lesson synthesis sheet
  - Lesson coding scheme
- **Framed within the perspective on teaching %**

3.1 Methodology

- **Videotapes of a sequence of 4 or 5 percentage lessons in each participating country**
- **Teachers were representative of good but not exceptional teaching practices in their country**
- **Collection of information about the local context**
- **2 analysing schemes: a lesson coding scheme and a lesson synthesis sheet:**

  based on live observations spread over one week of a series of lessons by members of the four country teams in each country (including videotaping)
- **Results were subjected to a Mann-Whitney U-test: to determine the influence of the approach**

3.2 Methodology: Lesson coding scheme

- **4 basic categories**
- **Mathematical focus:** underlying objectives of teacher's actions/decisions
  Subcategories: conceptual, derivational, structural, procedural, efficiency, problem solving, reasoning
- **Mathematical context:** conception of mathematics underlying the tasks
  Subcategories: real world fabricated data, not real world fabricated data, real world genuine data, not real world genuine data
- **Didactics:** teacher's didactic strategies
  Subcategories: activating prior knowledge, exercising prior knowledge, explaining, sharing, exploring, coaching, assessing/evaluating, motivating, questioning, differentiation
- **Materials:** teacher's and students' use of concrete materials
  Subcategories: answer book, blackboard, computer, calculator, coloured writing materials, display materials, game, pupil whiteboards, manipulatives, overhead projector, practical equipment, real world materials, worksheet, textbook
- **Coded for every episode of the lesson**

  1 if present and 0 if absent

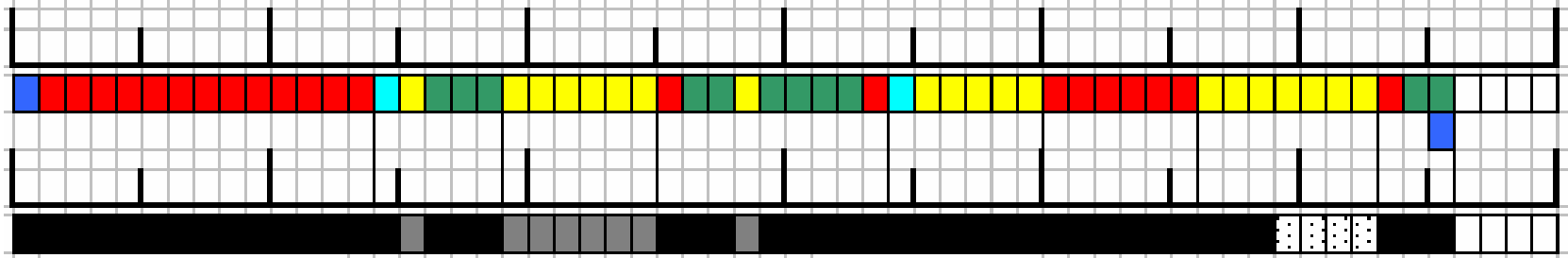
3.3 Methodology: Lesson synthesis sheet

- **Based on the work of Reusser** **(www.didac.unizh.ch)**
- **Brief details about the lesson involved**
- **Photograph of the classroom**
- **Two-dimensional timeline:**
  Pedagogic activities: subcategories: theory/conceptual development, working on problems/tasks, reporting solutions to problems/tasks, introducing a problem/activity, homework-related activities, task-related management, non task-related management
  Social activities: subcategories: whole class, group, paired, and individual activities
- **Narrative summary, linking a qualitative description of the lesson to the two timelines and to the categories of the lesson coding scheme**

M.E.T.E. summary sheet for video recorded lessons

Country: Belgium
School: Paridaens; Leuven
Class: grade 6; middle abbility
Teacher: Patrick Van Welde
Date: 05/11/2003
Topic and focus of lesson: The periphery of a circle (3)

Lesson Duration: 56'03"

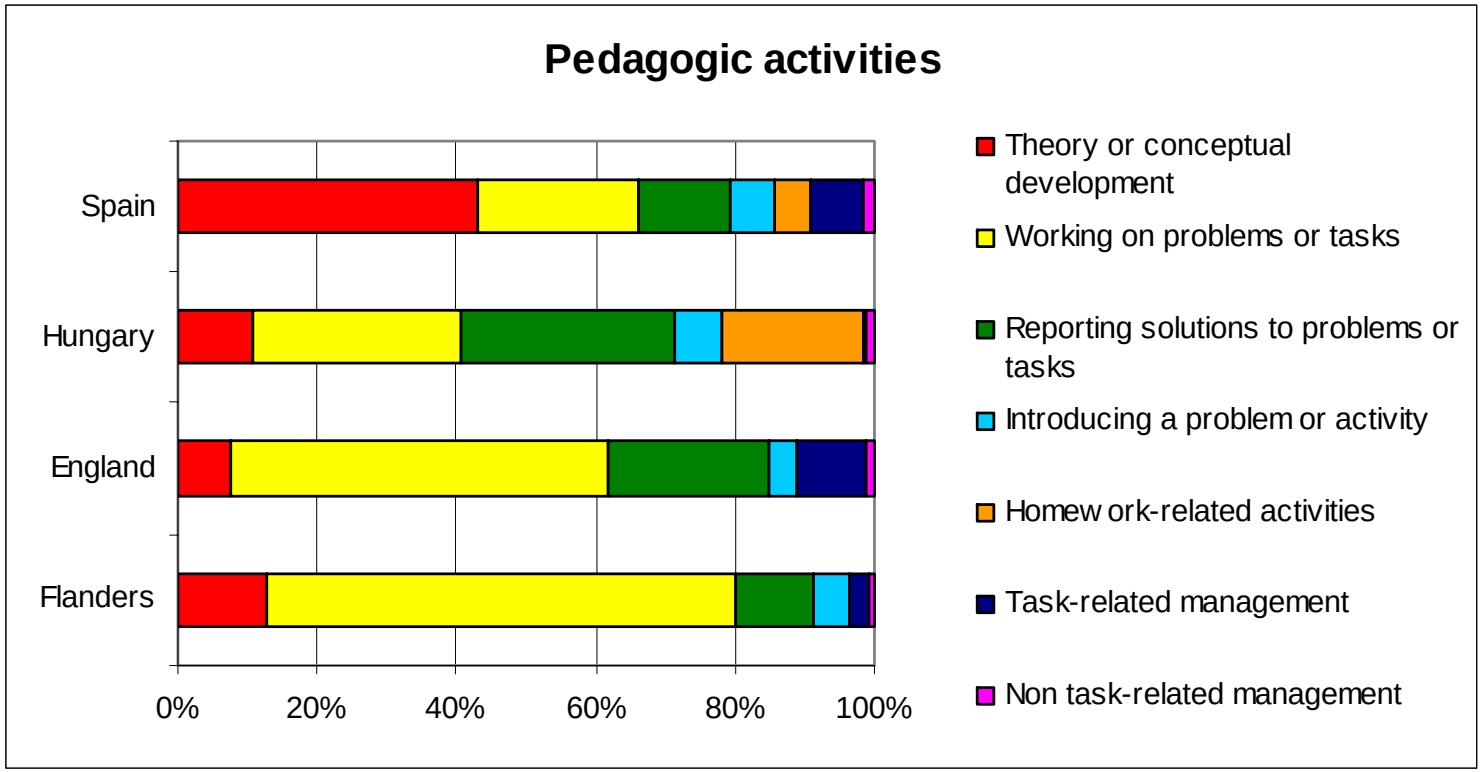


Pedagogic activity		
	=	Theory or conceptual development
	=	Working on problems or tasks
	=	Reporting solutions to problems or tasks
	=	Introducing a problem or activity
	=	Homework-related activities
	=	Task-related management
	=	Non task-related management

Social activity:		
	=	Whole class activity
	=	Individual activity
	=	Paired activity
	=	Group activity

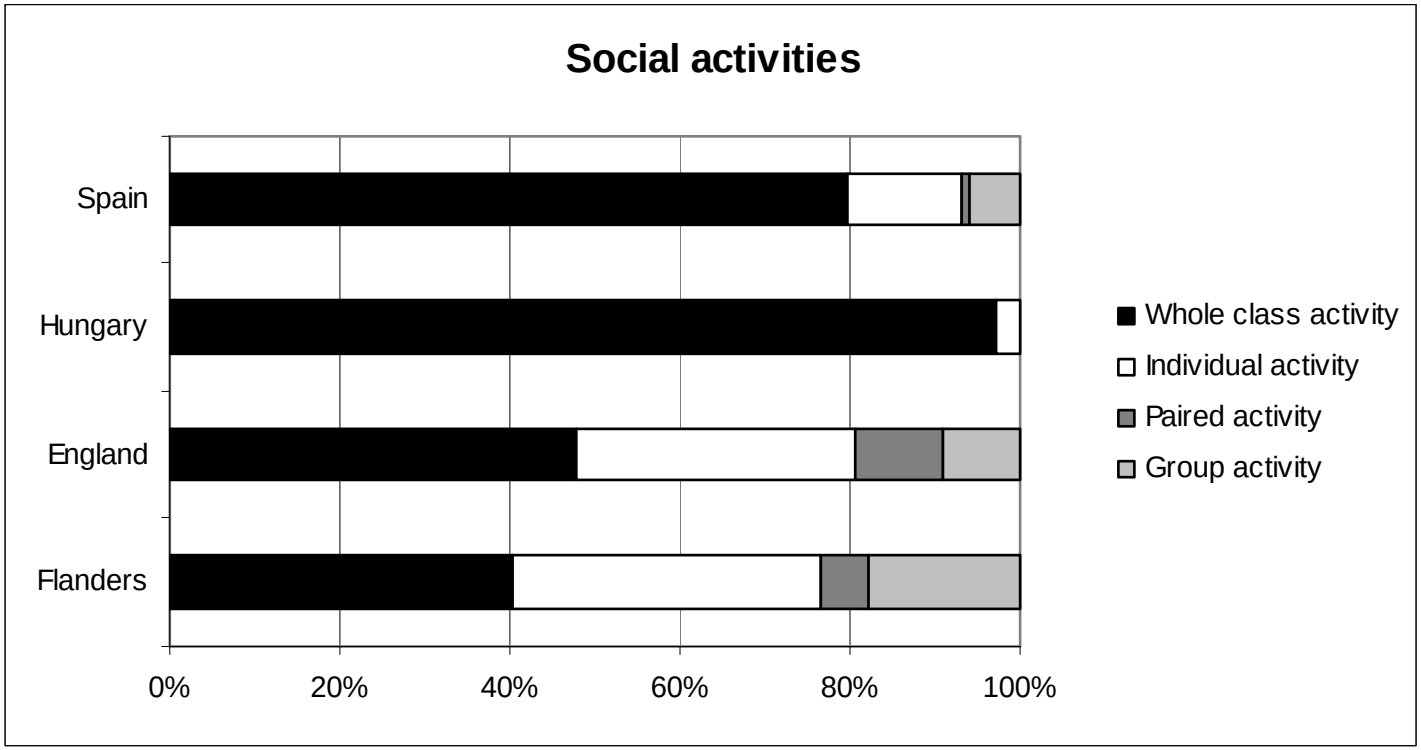
4. Results

4.1 Analyses of the lesson coding schemes



Similarities: A lot of time spent on: working on and reporting solutions to problems/tasks
Less time spent on: introducing a problem/activity, task-related and non task-related management

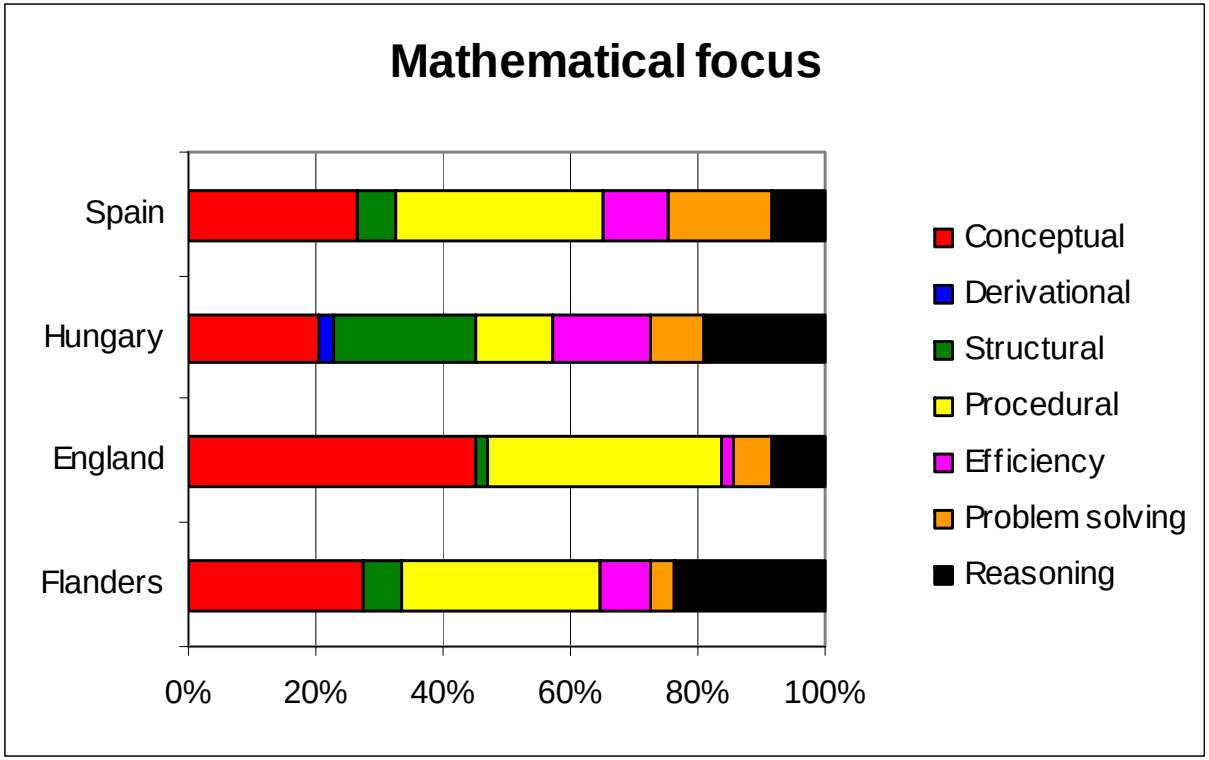
Differences: Great difference in theory/conceptual development, homework-related activities



Similarities: Whole class activities are dominant; working on individual activities have the second highest frequency

Differences: No paired and group activities in the Hungarian lessons.

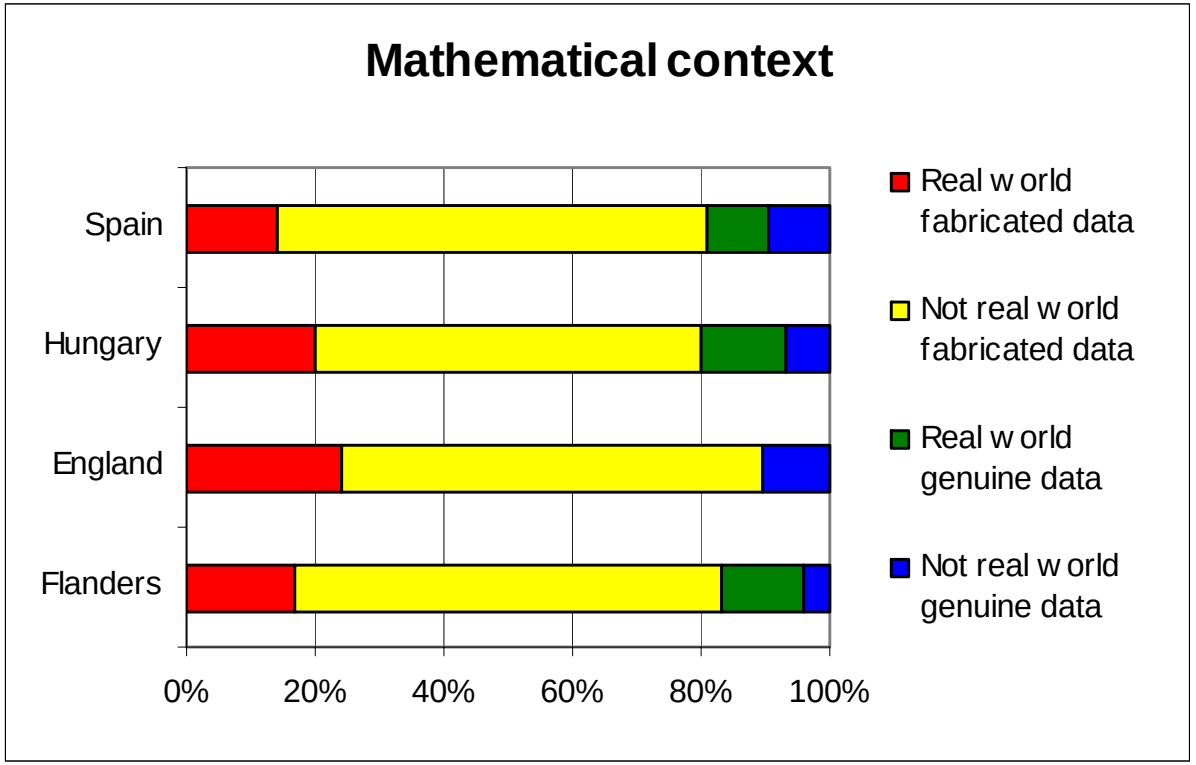
4.2 Analyses of the lesson coding schemes



Similarities: strong conceptual and procedural focus

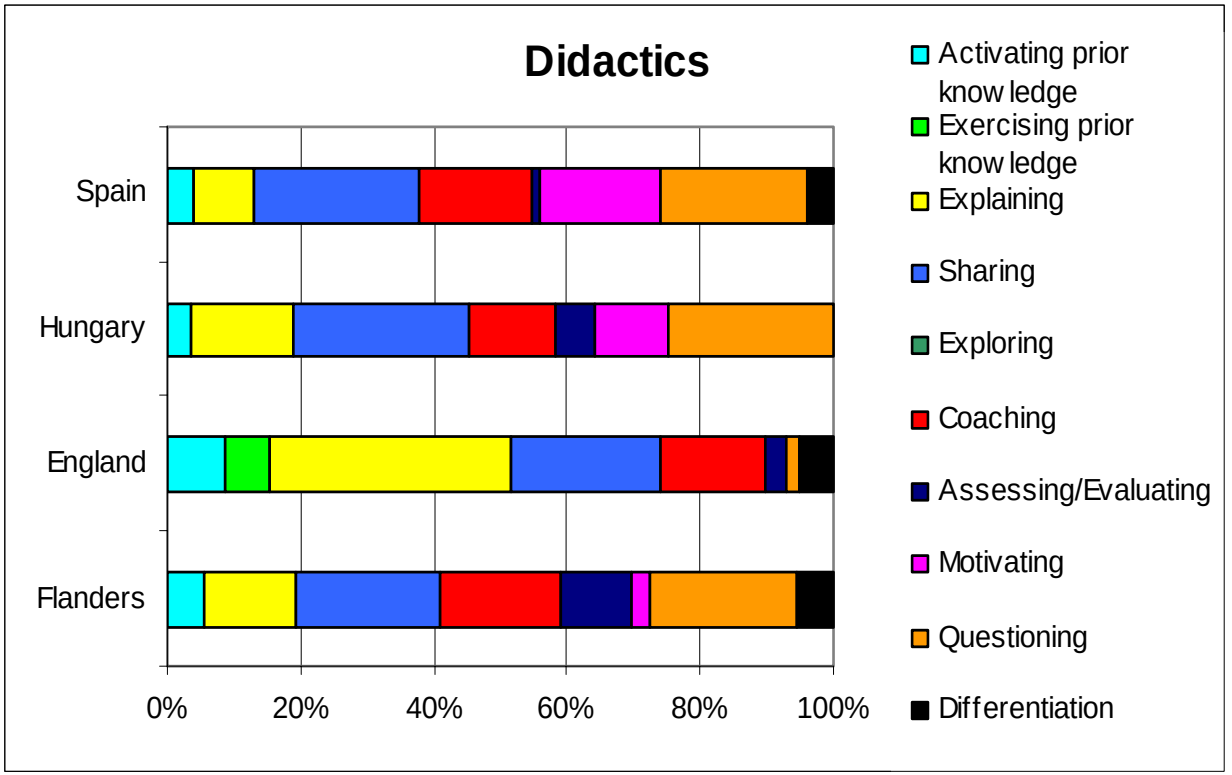
Differences: English lessons: significantly more conceptual ($Z = -1.96$; $p = 0.05$); significantly less efficiency ($Z = -2.23$; $p = 0.03$)

Hungarian lessons: significantly more derivational ($Z = -2.35$; $p = 0.02$), structural ($Z = -3.33$; $p = 0.00$), efficiency ($Z = -3.00$; $p = 0.00$), and reasoning ($Z = -2.43$; $p = 0.02$); significantly less procedural ($Z = -2.221$; $p = 0.03$)



Similarities: ± 2/3: not real world fabricated data; ± 1/5 real world fabricated data

Differences: No statistically significant differences between the different approaches to teach percentages

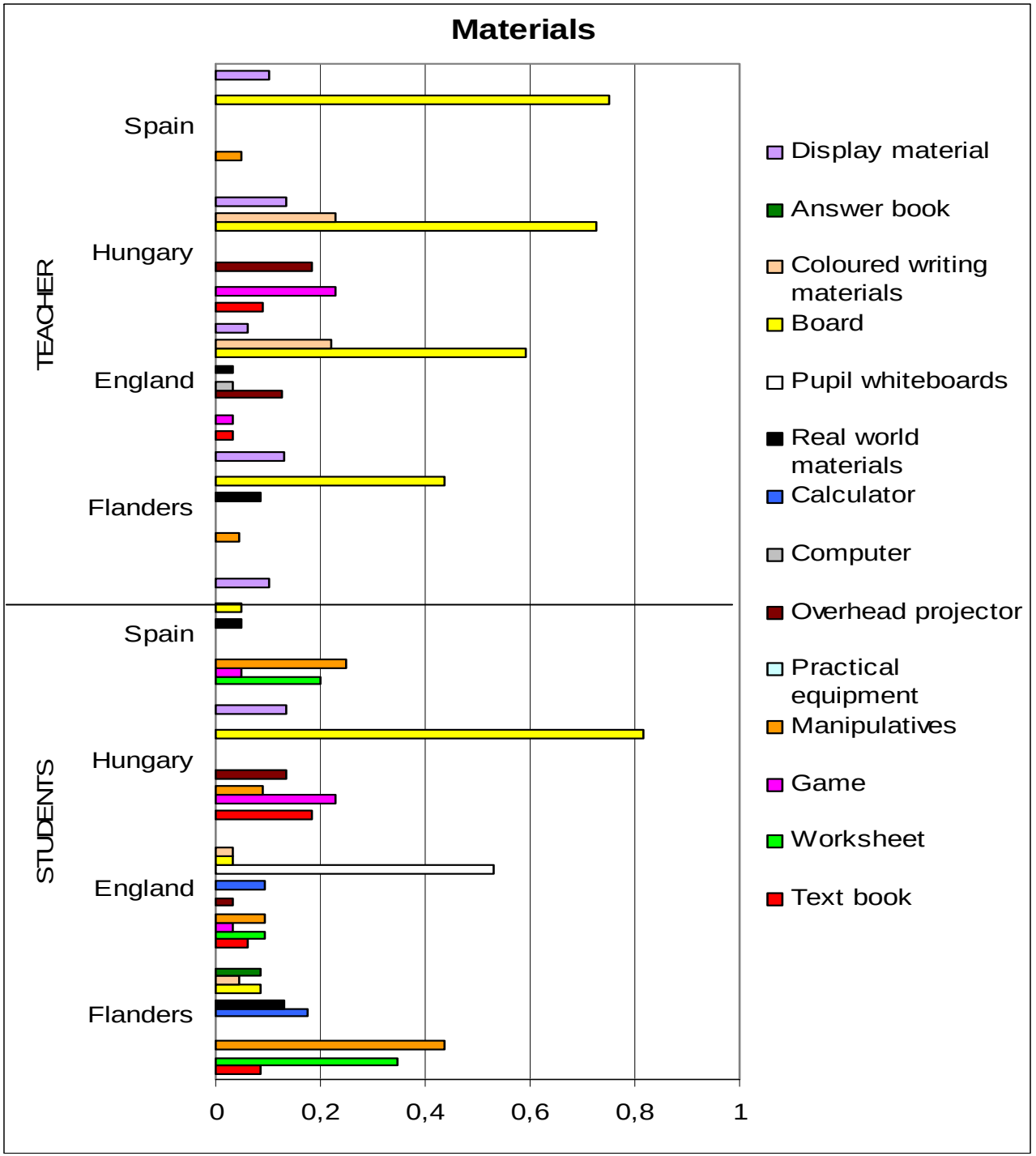


Similarities: 4 frequently used didactics: sharing, questioning, explaining, coaching

Differences: English lessons: significantly more activating prior knowledge ($Z = -2.01$; $p = 0.04$), exercising prior knowledge ($Z = -3.45$; $p = 0.00$), explaining ($Z = -2.53$; $p = 0.01$); significantly less motivating ($Z = -2.25$; $p = 0.02$) and questioning ($Z = -2.95$; $p = 0.00$)

Hungarian lessons: significantly less differentiation ($Z = -2.21$; $p = 0.03$)

Spanish lessons: significantly more motivating ($Z = -3.15$; $p = 0.00$)


Materials
TEACHER
Spain
Hungary
England
Flanders
STUDENTS
Spain
Hungary
England
Flanders
0
0,2
0,4
0,6
0,8
1
Display material
Answer book
Coloured writing materials
Board
Pupil whiteboards
Real world materials
Calculator
Computer
Overhead projector
Practical equipment
Manipulatives
Game
Worksheet
Text book

4.3 Framing the 4 approaches within the current perspective on teaching %

- **Objectives**
  - Computational goals
    - Strong procedural focus
    - Difference in the kind of procedures that were taught
      e.g., dividing the given amount by hundred to calculate 1% of that amount, and multiplying that result by the %;
      OR percentage-web: all % were related to 10%
  - Conceptual goals
    - Strong conceptual focus
  - Applicational goals
    - Mainly in the Hungarian approach: wide variety of tasks, each of which needed an appropriate solution method
      e.g., % and solution were given, and the students had to find the original amount
      OR exercises that contained a combination of increase and decrease of certain %

- **Conceptual aspects**
  - Strong conceptual focus
  - Main focus on “basic ideas” of %
    e.g., % always expresses something out of 100;
    100% = whole
  - In all approaches: different tasks reflect 2 types of situations:
    part of whole and whole +/- part
  - Different aspects of % that lead to deep understanding: hardly addressed
  - Hungarian approach focused more on the different aspects that lead to a deep understanding of %
    e.g., non-linear character
  - Flemish approach: to a less extent
    e.g., % describe a fixed situation

- **Didactic tools**
  - Everyday situations and students' informal knowledge
    - When introducing the concept
      e.g., sales
    - Only to a small extent: real world genuine data and real world materials
  - Relationship with other mathematical entities
    - Fractions in all approaches
    - Hungarian approach: decimals, degrees of a circle
      e.g., How many degrees equal 1% of a circle?
  - Models
    - Manipulatives in all approaches
      e.g., MAB-material, place-value cards
    - Models: ten by ten grid (F, H,S); pie chart model (E,H); arrow scheme (F,H)

5. Discussion

- **Scope:** small-scale videobased comparative study
- **Objectives:**
  not to evaluate or generalize the teaching of percentages in the participating countries
  rather to make an inventory of the variety in the different possibilities and traditions
- **Methodology:**
  development of instruments: time-consuming process
  use of multiple methods → qualitative data complemented quantitative data
- **Main results:**

	SPAIN	HUNGARY	ENGLAND	FLANDERS
Mathematical focus	Procedural Conceptual	Structural Conceptual Reasoning Efficiency	Conceptual Procedural	Procedural Conceptual Reasoning
Mathematical context	Not real world fabricated data	Not real world fabricated data	Not real world fabricated data	Not real world fabricated data
Didactics	Sharing Questioning Motivating Coaching	Sharing Questioning	Explaining Sharing	Sharing Questioning Coaching
Materials Teacher	Board	Board Game	Board Coloured writing materials	Board
Materials Students	Manipulatives Worksheet	Board Game	Whiteboards	Manipulatives Worksheet
Pedagogic activities	Theory/conceptual development Working on problems/tasks	Working on problems/tasks Reporting solutions Homework related activities	Working on problems/tasks Reporting solutions	Working on problems/tasks Theory/conceptual development Reporting solutions
Social activities	Whole class activity	Whole class activity	Whole class activity	Whole class activity Individual activity