Cultural differences in scholastic and non-scholastic environments: reasoning patterns and logical-linguistic questions in European and Chinese cultures

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Summary

This work analyses some similarities and differences between the different patterns of reasoning in different cultures. The epistemological and historic tools used are logical paradoxes which are traceable in Chinese and western cultures (problems of Aristotelian logic). The investigative tools are quantitative and qualitative. This work is placed within a framework of a vaster plan of research on the problems of teaching/learning in multi-cultural environments which are being confronted within the GRIM.

Our attention was concentrated on thought and the use of natural language which can transmit different ways of reasoning and expressing oneself. The experimental research, which investigates the deep sources of ways of thinking, can open new roads in the processes of teaching/learning of mathematics in every kind of school. This experimental work is subdivided into three parts:

1. *The first part concerns experimentation in class and the theoretic reference is that of the Theory of Situations [Brousseau, 1997]. The experimental data were analysed quantitatively [R. Gras, 2000] and qualitatively by means of the analysis of the protocols.*
2. *The second experimental part concerns the analysis of two cases and this analysis is of a type which is exclusively qualitative.*
3. *The third experimental part concerns the quantitative analysis of a group of Chinese students from Nanjing; ones who carried out the same problems done in Italy.*

By means of experimental analysis, the prevalent role of the didactic contract in standardising substantially the behaviours of the students has been highlighted. Comparing the interview protocols with the results of the questionnaires administered in the classes, one instead notes significant differences.

Key words: *Reasoning patterns, didactic contract, fuzzy thinking, logical-linguistic reflections.*

Introduction

The studies concerning the analysis of the schemes of reasoning are almost always orientated toward the teaching/learning of specific mathematical contents. The goal of this work is to present situations/problems typical of mathematical thinking, but with attention on the logical-linguistic problems.

This work is part of a research project which has highlighted:

1. the role of natural language in the development of mathematics in the history of thought. (Spagnolo, 1986, 2000, 2001, 2002);

2. the role of the history of mathematics as an instrument of observation and analysis of multicultural learning/teaching situations.

3. the role of fuzzy logic (an approach of the linguistic type) as an interpretive instrument of some problem situations in class correlated to "common sense" (Spagnolo, 2003; Ajello-Spagnolo, 2002). The main references are those of Zadeh, (as regards the fundamental considerations of fuzzy

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logic of the linguistic approach) and of Kosko (as regards the relationships and analogies identified between fuzzy logic and oriental thinking).

The most significant reference is the investigation, with historic epistemological instruments, of oriental patterns of reasoning in relation to European ones. Equally important is the epistemological analysis relative to the use of subtended logics in relation to natural languages.
All of this allows us to form the following hypothesis:

> ***H1: The differences and the analogies in the history of oriental and occidental cultures also have an equivalent in the differences and analogies between the patterns of reasoning found today in situations of teaching/learning of mathematics.***

To be able to falsify this hypothesis, we need the following paradigmatic references:

1. Historic and historic-epistemological analysis of mathematical thinking as regards the study of the different patterns of reasoning (deducing, conjecturing, demonstrating) in different European and Chinese cultures. This kind of analysis is conducted with the argumentations typical of history and epistemology and will be the basic reference for all the work. In some ways, it represents a possible point of view of ontogenetic development.
2. Experimental analysis of situations/problems in Palermo (Italy) by means of the Theory of Didactic Situations approach (Brousseau, 1997; Spagnolo, 1998).
3. Analysis of cases. This kind of analysis utilises the methodological instrument of the individual interview. The situations/problems discussed in class are the subject of the interview.
4. Experimental analysis of situations/problems in Nanjing (China) analogous to those tried in Palermo.

1.0 The works of reference: notes.

The principal reference for mathematics in Chinese education is that of the "Nine Chapters on Mathematical Procedures": this constitutes a canon[4] both for the construction of mathematics (1st Cent. B.C. – 1st Cent. A.D.) and for the teaching/learning of the same in the various historic periods. Among the most notable is the commentary of Liu Hui (263 A.D.) presented in the collection of the Mathematical Canons of the Tang Dynasty (618-907 A.D.). This canon of Mathematics was chosen to be included in an even greater reference regarding the revision of the classics of Confucianism.

1.1 Observations on the differences between the Chinese language and the Indo-European languages.

For the observations regarding language we refer to the following works: Chemla (2001), Needman (1981) and Granet (1988).

"*Chinese was able to become a powerful language of civilisation and a great literary language without having to worry about either phonetic wealth or graphic convenience, without even trying to create an abstract material of expressions or supplying itself with a syntactic armament. It managed to maintain for its words and sentences a completely concrete* ***emblematic value****. It knew how to reserve only for rhythm the care of organising the expression of thought. As if, above all, it wanted to liberate the spirit from the fear that ideas can become sterile if expressed mechanically and economically, the Chinese language refused to offer these convenient instruments of specification and apparent coordination which abstract signs and grammatical artifices are. It kept itself obstinately rebellious against formal precisions for the love of the concrete, synthetic* ***adequate expression****. Chinese does not seem organised for noting concepts, analysing ideas or* *conversationally expressing doctrines. In its completeness, it is constructed for communicating sentimental behaviours, for suggesting conduct,* ***for convincing****, for converting."* **(Granet, 1988)**

Words are nouns (ming) that refer to "existing things" (wu) in effective reality (shi). In fact, a word which means "old" does not exist; in compensation there is a great number of terms which illustrate different aspects of old age, with a full series of subtleties. The construction of the ideograms follows two complementary paths:

1. 80% of the ideograms are of the associative kind (Needman, 1981). They represent a sort of mental equations[4].
2. The practical rules for the construction of these ideograms can be synthesised by: "Rely on what precedes", "expand your conscience"[5].

1.2 From natural languages to logical linguistic aspects.

The passage from the analysis of language to the analysis of the logical-linguistic aspects is not difficult; in the historic Chinese tradition the main reference is that of the "School of Nouns", 370-310 B.C. It is exactly in this period that the logical-linguistic paradoxes were proposed which are taken into consideration in this work:

1. *"the distinction between that which you receive more and that which you receive less is the minimum of reception and distinction: that which in all beings is entirely received and entirely distinct corresponds to the maximum of reception and distinction"*
2. *"a white horse is not a horse"*

In agreement with the authors cited in this paragraph, the "School of Nouns" and the "Dialectics" are given a central role in the elaboration of scientific thought in China.
Kosko's hypothesis of fuzzy logic as a logic of reference of unknowing Chinese thought (at least until the end of the 1900s) represents one of the main references for this and other works. (Ajello, Spagnolo 2003). We analyse the first paradox using the following diagram. It is an attempt to give greater strength to Kosko's hypothesis.

The Paradox	***From the point of view of Fuzzy Logic***	***From the point of view of Bivalent Logic***	***From the point of view of Classical Chinese Culture***
The distinction between that which you receive more and that which you receive less is the minimum of reception and distinction: that which in all beings is entirely received and entirely distinct corresponds to the maximum of reception and distinction.	*A set A and the set not-A have in fuzzy logic an intersection which varies from a minimum to a maximum that depends on the possibility of receiving A and not-A and distinguishing A and not-A.*	A proposition of this kind does not enter in the Aristotelian syllogisms and is not found even in Hegel's dialectic.	It is part of the classical diagram of cohabitation of opposites as in the case of yin and yang[6].

2.0 Some reflections on "arguing, conjecturing and demonstrating" in Chinese Culture with relation to Occidental Culture.

[4] "A third of the characters is composed of semantic combinations of two or more pictograms, which form those that we call composed by association. Thus *fu*, wife, is composed of signs for woman, hand and broom; … we therefore have a sort of equation: *li (field)+tien(strength)=nan(man). Such equations constitute a semiconscious mental foundation for whoever is acquiring familiarity with the language."* (Needham, 1981, pp. 35-36, vol. I).
[5] Both of the rules come from two similarly named works (200 B.C.) that represent a historic epistemological reference for philosophy and for deduction and demonstration in mathematics.
[6] "The symbol yin-yang is the emblem of nuance, it represents a world of opposites", it also represents the instrument of fundamental reference of Chinese thought in all fields of knowledge both humanistic and scientific. We can compare it to the organisation in categories of Aristotelian logic.

This paragraph analyses in a schematic way some substantial differences found in the history of Chinese thought and in the history of western thought.
In the comparative analysis of science in pre-modern China and the west, Geoffrey E.R. Lloyd (2001, pag.574) says that, "The aspirations of ancient Greek tradition represented by Euclid, which proposed deducing all mathematics from a single set of indemonstrable but evident axioms were not shared by the Chinese at least until the modern age. In China, as a matter of fact, the goal was not axiomatic-deductive demonstration, but gathering unifying principles from all of mathematics."

2.1 The algorithm as fundamental element of arguing and demonstrating?

In Chinese mathematical thinking the main reference is the algorithm. It plays a central role in the Canon of mathematics and also represents a tool for algebraic demonstrations. In the solving of a problem which foresees the rule of three (substantially it has to do with the uniqueness of the fourth proportional), for example, the initial data are considered as conditions (if… then) "if I have a certain quantity of silk then I have spent a certain amount of money", and also the solution of the question "What quantity of silk can I buy if I have a different amount of money than previously?" can express, in the same way, (repeating the previous condition) "if I have a certain quantity of silk (which I don't yet know) then I have spent a certain amount of money (which I know)". Thus the variable is identified and with the process of reduction to unity (by means of the properties of the proportions, in our way of proceeding) the unknown value is obtained. The process for the solution is standard and is therefore an algorithm. Demonstrating the validity of that reasoning means demonstrating the correctness of the procedure (use of the properties of the operations) in the steps of the algorithm.

The algorithm is a combination of an iteration and of chosen 'conditionals'.

The chosen conditional is a first interesting element of the pattern of reasoning:

1. Iteration
2. Conditionals (If…then…)
3. Assignment of variables

The following table attempts to find analogies and differences between the meanings that the algorithm assumes in the two cultures.

	From the occidental point of view:	***From the oriental point of view***
Intuitive algorithm	Procedure	Procedure. Research of fundamental algorithms as reference.
Formalised algorithm	Algorithm: 1) Effectiveness, actually feasible by an automaton. The automaton must be able to recognise the minimum parts of the description of the algorithm (accepting the language in which the algorithm is written; the well formed sentences are called instructions). 2) Finiteness of expression: finite succession of instructions. Cycles, conditions, jumps. 3) Finiteness of the calculation: in the concept of algorithm there is usually included the condition of termination of the procedure for any situation of initial data within a certain domain. 4) Determinism: at each step of the execution of the procedure one and only one operation must be defined and successively carried out.	A paradigmatic example is the rule of three: the rule of three rests on the *"quantity of that which one has"* and on the pair constructed from the *"lü of that which one has"* and of the *"lü of that which one is looking for"* to give rise to the *"quantity that one is looking for"*[7].
Deterministic algorithm	Condition 1 is inalienable. The others give rise to different types of algorithms. If 4 is missing, the algorithm is called non-deterministic.	Research toward analogies of valid algorithms for classes of homogeneous

[7] The qualification of the *lü* highlights that the quantities are defined one in function of the other. However, it also highlights the possible role of unknown. As already observed in note 3, the reasoning pattern induced from natural language brings the Chinese to implement some resolutive strategies of an algebraic type (Spagnolo, 1986, 2002). In other situations the role of the unknown is expressed by its position in perfect agreement with the approach with the rods.

		problems. Reference to the algorithms as true and real models.
Probabilistic algorithms	Approximate, probabilistic, NP-complete algorithms (if there exists a polynomial algorithm able to confirm whether or not this is effectively the solution of the problem), algorithms that stop after a number of steps which grows exponentially.	Fuzzy algorithms?

Human thought is based on heuristics and not on algorithms as understood in the formalised western sense. Human decision makers formulate their decisions on subjective heuristics. These heuristics are founded on personal experiences, on (adductive?) extrapolations and on probabilistic (fuzzy?) evaluations of the costs and benefits with the goal of arriving at the least risky decision possible in the presence of the scarcity of available objective data.

2.3 What are the stable reasoning patterns in Chinese culture?

Each argument is concluded with phrases of this type "from here the result".

The algorithm is seen as an instrument for demonstrating the precision of an argument.

If there are successive divisions, in geometry for example, the algorithm is declared correct only when it is demonstrated that in the process followed the quantity not yet dealt with tends toward zero. (Recall Archimedes' method of exhaustion)

One stable reasoning pattern is the following:

"Making homogeneous and making equal": (from the commentaries of Liu Hui, 263 B.C. (Chemla, 2001, pg. 142))

"*Multiplying to separate them, simplifying to unite them, making homogeneous and making equal so that they can communicate: how could these not be the fundamental points of mathematics?"*

The demonstration is not only the correctness of the reasoning. "Making equal" and "making homogeneous" which represent concrete indications on algebraic manipulations enter into play, but also strategies of reference for then being able to concretise the correctness of the reasoning through the algorithm.

An interesting example of the "Making homogeneous and making equal" is that of the rule of three (from the commentaries of Li Chunfeng, 656 B.C. (Chemla, 2001, pg. 142). This algorithm once again is an operation which 'makes equal' and 'makes homogeneous' (in the reduction to unity). So, the rule of three, as a fundamental algorithm, is the parallel in western culture of the postulate. The fundamental algorithm can combine itself several times always arriving at a sure argument.

As Liu Hui observes, applying such algorithms, the values shouldn't change and this guarantees the truth. Therefore, particular attention must be given to the examination of the algorithm on the classes of problems to be able to highlight its correctness.

One strategic objective of the Chinese was that of correlating the different processes of calculation employed in diverse areas of mathematics for demonstrating their unity. (research on invariants).

Needham (1981) maintains that after 1700 the two cultures fused, while P. Engelfriet (2001) maintains that this process was longer and perhaps is still being carried out.

In this table a diagram of comprehensive reference on some significant differences between the two cultures is presented with respect to the cognitive instruments of the deducing. Naturally, such a table is still a work tool to be perfected and with interesting open problems to be discussed again.

	Scientific technological revolution.	***Inferences***	***How one knows***	***How to confront, today, the question from the viewpoint of the science of complexity***
Occident	1600 scientific revolution: bivalent logic ***Tool of bivalent logic:*** *a priori knowledge of the possible scientific-*	1) Inductive 2) Deductive 3) Adductive	Categorical diagrams (Aristotle) Manipulations of algebraic formulas out of context for constructing abstract modelling activities to foresee phenomena in a deterministic way	***Semiotics?*** ***Systemic approach?***

	mathematical and technological modelling activities.			
Orient	XXI century scientific revolution: fuzzy logic? ***Tool of fuzzy logic:*** a posteriori knowledge of possible scientific-mathematical and technological modelling activities. Logic of analogies? Logic of correlations?	Semiotic inferences. 1. If…then… 2. Adductive 3. Making equal 4. Making homogeneous 5. Algorithm - Interaction - Conditionals (If…then…) 6. Assignment of variables	Consciousness by way of one's whole body: modern neuro-physiological theories, manipulations of algebraic formulas always referred to a context (as in tradition) Embodiment?	The computer-based demonstration (for example the theorem of the four colours). The technological applications of fuzzy systems. Possible tool of unification of the consciousnesses: neuro-physiolocial, consciousness by means of one's own body, overcoming of the divisions of the mind - body heritage of Cartesian philosophy, holism .

There exist, effectively, many analogies with western thought at least with respect to the recent developments of the neurosciences. Presumably, the most important reference is that of acquisition by means of Models and the Hierarchization of Models which correspond to Deducing for organising arguments.

Such Hierarchization is strictly tied to linguistic organisation:

Good order depends entirely on the correctness of the language which according to the point of view of the authors is in agreement with the fuzzy approach:

Indications for correct argumentations according to Chinese thinking	***Interpretation according to the Fuzzy thinking of the passages by topic***
Correct designation and correct predication: *these are the practical indications which are indicated by numerous Chinese intellectuals*[8]	**Correct Fuzzy relationship, conforms to concrete situations: *translation of the linguistic rules in inferences fuzzy sets on fuzzy sets.***

3.0 How were the situations/problems chosen?

Each situation foresees a possible reasoning pattern, but does not exclude others. (The questionnaire is in appendix 1)

In questions 1 and 4 the term "prove" is used deliberately and in 3 "demonstrate" is used. This is because the first and the fourth questions lend themselves more to processes which induce proofs, to empirical attempts. The third, instead, foresees reasoning by deduction, in any case, whatever the technique may be (by means of the representation of the possible cases or not) to arrive at the solution. The fifth necessitates reasoning by "exclusion of cases" which comes closest to a reasoning to the impossible. Question 2 is a paradox of the traditional Chinese culture which dates back to the "School of the Nouns" (370-310 B.C.), that plays on the linguistic ambiguity relative to qualities and it lends itself very well to analysing the different oriental (Chinese) and occidental points of view.

The argumentations for confronting questions such as the ones proposed are closer to natural reasoning than to mathematical demonstration. The analysis of the different discursive forms and the different levels of organisation of the argumentations produced by the young people in the sample chosen, offer, however, the possibility of distinguishing and comparing different reasoning patterns and making some instructive observations about different behaviours.

The only implicit bond to which the argumentations are subjected because they are considered acceptable is the pertinence.

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8 In reference to Tao, ancient Chinese philosophy, there are some analyses of a Complex of very close ideas which could follow the following pattern:

1. Order
2. Totality
3. Responsibility
4. Efficacy

4.0 Presentation of the experimental work in the Italian classes

Five situations/problems have been formulated with the primary objective of identifying different patterns of reasoning.

This work was carried out at the state Scientific High School "S. Cannizzaro" in Palermo.

The classes involved were: one third year (16 year olds) and one second year (15 year olds).

The young people of the third year had already confronted the question of Aristotelian syllogisms and therefore they expressed themselves more suitably, where the students of the second year were, in any case, able to solve the questions (the percentage of the questions solved was very similar). Everyone was able to use the language of set theory correctly.

The methodology followed and the analytical tools used:

- The questionnaire was distributed to the young people of the two classes by the same teacher (Prof. Ajello) and they were given the same information and the same clarifications of the questions. The time available to them was 90 minutes.
- The protocols were collected and analysed on the basis of previously formulated a priori analysis and the data was drawn up with the CHIC, for the implicative analysis, and with the SPSS for the factorial analysis.

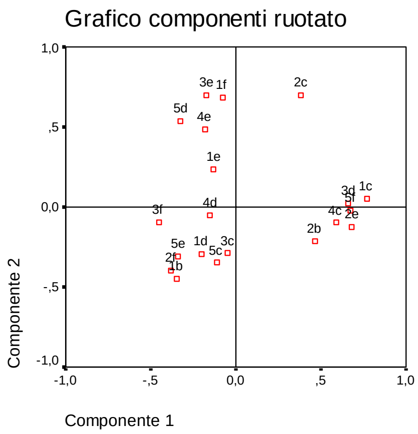
4.1 The quantitative analysis: implicative and factorial

4.1.1 The factorial analysis.

The variables relative to the "missing" and "incorrect" answers were eliminated from this analysis just as in the implicative analysis and this was done to better highlight the patterns of reasoning.

The information is 31.86% important with respect to the number of variables (29).

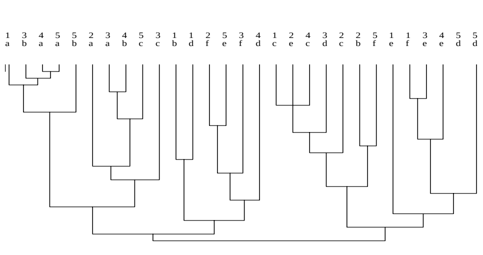
Two factors can be identified. The first has to do with the group of variables on the right (2b, 4c, 2e, 3d, 1c, 5f) which refers to the richest reasoning (inductive, deductive, Aristotelian, etc…) and more elaborate reached by a certain group of the young people. Analogously the same variables are in a circular implication in the implicative analysis (implicative graph follows). The other group is represented by the remaining variables (except 2c) and corresponds to correct behaviours but is not always explained in an exhaustive way.



A separate discussion regards variable 2c which corresponds to the ability to deduce contemporaneously with opposite arguments that arrive at opposite conclusions (the proposition of question 2 can be, at the same time, true and false, a logical-linguistic paradox of Chinese culture).

4.1.2 Implicative analysis and of the similarities

The following graphics are more meaningful, other graphics have only confirmed how much is already deduced from the following two reported.



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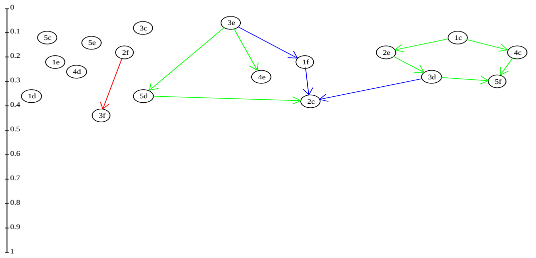

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4.1.3 General considerations on factorial analysis, implicative analysis and of the similarities

From the circumstances one deduces that: 3f and 2b were the most frequent responses and they both correspond to a pattern of reasoning ascribable to the rules of deduction of Aristotelian syllogisms.

From the graph of the similarity tree one notes that the correct, but not explained, answers are grouped either with the attempts or with the incorrect or missing answers. From the implicative graph one notes that the same answers, above, are in relationship among themselves but never with others of other types. It is thus appropriate to exclude from the graphs answers 1a, 2a, 3a, 4a, 5a, 3b, 4b, 5b.

Looking at the implicative graph, the most important considerations are the following:

2f → 3f whoever recognises a paradox in the proposition of the 2nd question responds to the 3rd question with Aristotelian reasoning patterns.

* from answer 3c (answers correctly using tables and matrices in the reasoning patterns) three implications branch 3c → 5d → 2c ; 3c → 4c ; 3c → 1f → 2c.

One possible interpretation: who makes use of tables or matrices is able to do inductive reasoning and confront a paradox even with contrasting patterns and is able to produce proofs for the impossible and correctly use the meaning of the division by distribution.

* 1c is another importance crux, in fact there are two significant chains of implications: 1c→ 2e → 3d → 2c e 1c→ 2e → 3d → 5f → 2b

literally: whoever solves the 1st question, using induction correctly, (which appears the most difficult from the circumstances) manages to leave the traditional patterns of deduction and in question 2 proves that the proposition is true interpreting it like "a white horse is not just any horse", but he is also the one who correctly uses inclusion of the sets to show the structure of an inference graphically and still uses matrices with arrows that indicate the relationships.

Summary of the results

Among the possible reasoning patterns that the students of the sample used, the most difficult to support and was the tied to induction while the most often correctly used ones were the 'chains' of deduction also with graphic representations of the inferential structures.

This, basically, is a quite foreseeable result because of the set-up of the study of mathematics in schools, but perhaps a part of all disciplines has always favoured inferences and therefore deductive reasoning, neglecting the importance of training the students to utilise alternatively induction – deduction – induction – deduction.

Perhaps the second result, which involves the different ways of reasoning, is more interesting: the ability to utilise graphic representations, combinatory methods, accepting, without much surprise, the possibility of encountering a paradox, encourages the correct use of syllogisms, preparing the way for a conscious use of mathematics demonstration.

Giving a look at the characteristics of parallel thinking (pt) and serial thinking (st), one can compare the ability to use multiple ways of thinking, even accepting contradictions to pt, while the ability of the conscious use of syllogisms can be compared to st. So, the alternating pt – st – pt – st – pt can correspond to moments of creativity in which one result is seen from many different points of view and moments of systematisation in which one reconstructs and explains a result.

4.2 Interviews with two Chinese: qualitative considerations

Two interviews were carried out on the sketch of the situations/problems done in class.

Tong (born in Canton 1954) went to Chinese schools until the upper secondary experimental school, but did not however complete his studies. He moved to Palermo in 1978 and obtained his Italian middle school diploma in 1985. Currently he manages a Chinese restaurant in Palermo.

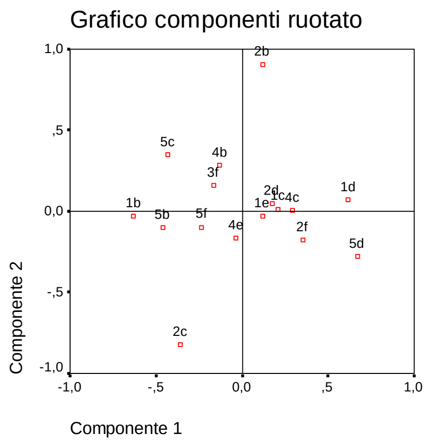
Jouzou (born in Palermo in 1986?) is currently attending the last year of an experimental study course European High School. He has studied Latin, Greek, philosophy, mathematics, etc., and he considers himself to be culturally Italian. His awareness of the Chinese language and culture has been by way of his parents who are both Chinese. Tong is Jouzou's father.

4.3 The experience in the Chinese classes.

The experience was conducted in September 2003 in some classes of the upper secondary schools of Nanjing and with some students of the 1st year of university. The questionnaire was the same one given to the Italian students. The a priori analysis of the behaviours was shared by Doctor Zhang Xiaogui and translated into Chinese. The data collected were tabulated following the same scheme used in paragraph 3.1. Important results were not discovered with the implicative analysis of the variables.

My exam subjects were 12th grade high school students and first year university students. Students chosen were representative. There were 105 students, 65 high school and 40 university, who joined the exam. The questionnaires received back included 52 from the 12th grade and 30 from the university students.

5.1 Factorial analysis.



The variables relative to the "missing" and "incorrect" answers were eliminated from this analysis as in the implicative analysis and this to better highlight the reasoning patterns.

The information is 25.3 %, important with respect to the number of variables (20).

A small number of variables appears because the answers were concentrated only on some variables. The most significant result is that relative to the variable 2c which differentiates itself form all the others as in the case of the Italian sample. This analysis did not give other relevant information.

4.3.2 Supplementary variables.

To be able to better analyse both the Italian and Chinese data, we have introduced supplementary variables. These variables, which we called "***europ***" e "***fuzzy***", characterise different patterns of reasoning:

"europ" ***Reasoning patterns typical of western scholastic[9] education and especially in Europe.***	***"fuzzy"*** ***Reasoning patterns typical of classical Chinese[9] mathematics education.***
1c, 2b, 2f, 3f, 5d, 5e	***1e, 1f, 2c, 2e, 3d, 3e, 4d, 5e, 5f***

These variables were selected on the basis of epistemological and historic-epistemological analysis, but also on the basis of preceding experimental works (Spagnolo (2003), Ajello-Spagnolo (2003)).

In appendix 2 we have inserted the experimental data with regards to these analyses. This foresees that in the initial table with the addition of the supplementary variables (profiles of student type) it is done there. At this point the variables will be the students and the two profiles “europ” and “fuzzy”. This allows a new vision of the situation.

4.3.3 Considerations on the analysis of the data relative to the Chinese sample.

Both in the implicative analysis of the variables and in the factorial analysis one observes a well defined division of students who imply or get close o the two supplementary variables in a distinct way (see appendix 2). For this reason, we can, without doubt, confirm that these two patterns of reasoning are present today contemporaneously both in occidental and in Chinese schools. This is an important result that is not completely identifiable a priori with only epistemological analysis (which instead has clear references to education in general). Fuzzy reasonings are always present even in western culture in agreement with a wide body of literature which confirms the fact that spontaneous conceptions on deducing and demonstrating are not always tied to bivalent logic. Thus, the attitudes which refer to bivalent logic are well present, in the answers of the Chinese young people. However, on the other hand, the presence of the deductive type of reasoning in the Chinese school and of the fuzzy type in the Italian school causes us to take into consideration the role of scholastic culture

Further experimental works are in progress to be able to better understand the role of the patterns of spontaneous reasoning in the two cultures.

5.0 General conclusions and future perspectives

Following is a comparative analysis between the different behaviours which emerged from the enquiry of the questionnaire in class in Italy and China and the interviews. The different points of view include all the casuistic of the answers and are reinforced by the historic-epistemological analysis of mathematics in the two cultures with particular reference to deducing and demonstrating.

Questions	Prevalent behaviours in the protocols of the interviews (PC)	Prevalent behaviours in the experimental results in Italy (I)	Prevalent behaviours in the experimental results in China (C)
1	***Heuristic approach for attempts and errors. Research of an algorithm as a tool of formalised demonstration.***	***Inductive reasoning: finite chain of conjunctions.***	***Experiment and induction. Proof and intuition.***
2	***Request of a concrete context to analyse the adequacy of the proposition in hand.***	***Use of Venn's diagrams for deduction (the proposition thus ends up false).***	***Proof of the truth and falsity of the propositions.***
3	***Measure of the conformity of the affirmation at hand with the premises. More care of the analysis of the text. Use of tables or matrices.***	***Use of Venn's diagrams for deduction and a correct interpretation of the syllogisms. Deductive processes in N.L.***	***Correct us of the reasoing patterns of the syllogisms.***
4	***Organisation of the data for the research for conformity with a model (diagrams, previous idea, analogous situations)***	***Organisation of the data for the analysis of all the possible cases. Use of division by distribution. The pigeon hole principle.***	***The problem is not recognised as referable to a known pattern of reasoning. It is not solved.***
5	***Reasoning of the combinatorial type with representations by tables. Analysis of all possible case to encourage the renewal***	***Use of contrapositives and therefore of reasoning to the impossible in N.L. and with the help of double***	***Use of counter contrapositives and therefore reasoning to the impossible in N.L.***

	of the model.	***entry tables.***	

Three important results can be identified:

1. Deductions both for demonstrating that proposition 2 (the Chinese paradox, variable 2c) is true and for demonstrating that it is false are present in high percentages in the two samples examined (I) and (C). This brings to light that fuzzy reasoning is present in equal measure in the two scholastic populations (Ajello-Spagnolo (2002)).
2. With respect to question 4 (which refers to the so-called “the pigeon hole principle”) most of the young people of the Chinese sample did not answer while the percentage of the young people in the Italian sample that did not answer is insignificant. The question is not referable to patterns of pre-organised reasoning (it does not have reference models). From the point of view of classical Chinese mathematics education, this constitutes a problem in the moment in which one looks for the reference to a pre-established model for confronting an “analogous” situation.
3. While numerous similarities are come across between the two samples examined in a situation of stated didactic contract (I) and (C), the answers in the protocols of the interviews (PC) turn out to be more varied where one sees greater cultural influence in the absence of a didactic contract.

The initial hypothesis needs, therefore, to be reformulated in the light of the role that that didactic contract plays in confronting the situations/problem proposed.

In the preceding work, Ajello-Spagnolo (2003) where the investigation of the Chinese scholastic sample was not present, the differences between the patterns of reasoning in the two cultures were more marked.

Other developments:

1. The analysis must continue with other experimental works through interviews of Italian young people and Chinese young people (who live in China) outside of the didactic contract.
2. Conjecturing and demonstrating (here we refer to mathematical demonstration) have not been confronted in the experimental phase. We are thinking of preparing a series of problem situations for a careful investigation.

Open problems:

1. Up to what point does the didactic contract also impose reasoning patterns thus intervening in the deep-rooted logical-linguistic structure?
2. What is the range of action of the didactic contract in the case of didactic situations in multi-cultural environments?

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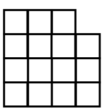
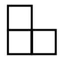
Appendix 1

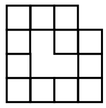
Questionnaire on the abilities "to argue" problematic situations

1. A chessboard of 2^n x 2^n cells is given. You remove a cell in one of the four angles, for example:





Is it possible to cover the whole chessboard with pieces of this type?
Suggestion. To reason by induction, you put the piece from three in the centre:
How does it proceed?



1a) Solution
1b) It motivates the proposed solution

2. "A white horse is not a horse." Are we able to declare this proposition true or false?
   2a) Solution
   2b) It motivates the proposed solution

3. The premises are given: "All the adults can vote. Sabrina is of age. All those people who have a driver's licence are of age." It considers the following affirmations:
   a. Sabrina can take the licence
   b. Who is not of age doesn't have a licence
   c. Who doesn't have a licence is not of age
   d. Sabrina can vote

   It shows that three of them are true and only one is false.
   3a) Solution
   3b) It motivates the proposed solution

4. In a class there are 30 pupils. In the dictation, all have made at least one mistake.
Alex has made 13 mistakes and all the others have done less than he did.
Prove that there is at least one group of three pupils that has made the same number of mistakes.

4a) Solution
4b) It motivates the proposed solution

5. Mario, Benedetto and Giovanna are the first names of three young people aged 14, 16 and 17. Rossi, Bianchi and Verdi are their last names. The order of the first names can correspond or not to correspond to that of the last names, and it is not known to whom the ages belong. Knowing that: the girl Rossi is three years older than Giovanna and the young Verdi is 16 years older, find the complete name of every pupil and his age.

5a) Solution
5b) It motivates the proposed solution