

Curriculum Issues in Statistics Education

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In modern societies, governments are increasingly making decisions based on data, decisions about where to build roads and schools, policy decisions about health care and employment, operating decisions about effective management. As individuals, citizens are empowered perhaps more than ever in any society to make countless decisions affecting education, health, money, careers, even the way they are governed. Citizens also have more access to data, from public opinion polls to medical research findings to millions of web pages with content of widely varying veracity. Faced with such freedoms, a barrage of information, and inevitably the uncertain consequences of their choices, an understanding of statistics and probability is increasingly important for every citizen and for every potential government leader. For the functioning of our society, pre-K-12 education must ensure that students learn to reason about and with data while taking into account uncertainty.

Education systems are responding to the increasingly important role of statistics by including statistical strands in their curriculum, usually in the mathematics curriculum along with number, algebra, and geometry (see for example, NCTM, 2000; Singapore, 2001, 2005; MIUR, UMI, SIS, 2003 in Ottaviani, in review). However, what is known about how statistical ideas might developed to promote understanding is rarely linked to curriculum. Often, the same concepts – mean, median, making graphs – are taught over and over again across the grades, with little variation in presentation or expectations for growth in understanding. Researchers are finding that students lack the ability, at all levels, from elementary to tertiary, to reason and think statistically (see for example, Ridgway et al, in review; Matis et al, in review, Garfield & Ben-Zvi, in press). Many report research and experiences indicating that students often master technical skills but are unable to use these skills in meaningful ways.

Questions that should frame the discussion include: What is important to teach, when should it be taught and how? How do we carefully structure the curriculum? What do we know about when concepts should be taught and how? How can we make statistics education more inviting? What do we know about teaching and learning statistics and what do we need to know? How does research link to practice? One source that considered these questions was the IASE Roundtable on Curricular Development in Statistics Education held in Lund, Sweden on 28 June to 3 July 2004. The Roundtable provided a forum for 26 participants from nine countries to consider aspects of the statistics curriculum from primary school to the tertiary level (Burrill, in review). Another source is the 2006 National Council of Teachers of Mathematics yearbook on *Reasoning and Thinking with Data and Chance* (Burrill, in press). And yet another source is the A Curriculum Framework for PreK-12 Statistics Education created by the American Statistical Association's committee on Guidelines for Assessment and Instruction in Statistics Education (GAISE) (Franklin et al, 2005).

The role of frameworks as a guide to thinking about curriculum development in statistics, assessment, conceptual understanding, and teachers' practice is a theme that emerged in the Roundtable papers. The case was made for beginning with a clear and well-articulated framework that could inform the development and analysis of how understanding unfolds, along with a caution that learning is not linear and that it might be more appropriate to think about curriculum as a set of cycles (Begg, in review).

Given that statistics is often taught as part of the mathematics curriculum, one of the issues a framework must address is the difference between mathematics and statistics. Scheaffer (2004) argues that mathematics is about numbers and their operations, generalizations and abstractions, spatial configurations and their measurement, transformation and abstractions. Mathematics is about logical reasoning, patterns and optimization, proof and abstraction. Statistics, is also about numbers but numbers in context and about variability and the purposeful design of studies trying to understand, measure, and describe real world processes. According to Scheaffer, the real value of statistical methodology lies in its usefulness in solving a problem of interest, not in any generalizable properties that can be proved theoretically.

The GAISE framework defines three levels of understanding; a level can be associated with grade level bands but the previous level of understanding is necessary before moving to the next. The

framework is structured so the depth of understanding and sophistication of methods used increases across the levels A, B, C. Statistical problem solving is presented as an investigative process that involves four components: Formulate questions; collect data; analyze data; and interpret results. Table 1 shows how these ideas might play out in each level for formulating questions.

Table 1 (GAISE)

| Process Component | Level A | Level B | Level C |
|---------------------------|--|--|--|
| Formulate Question | Beginning awareness of the statistics question distinction Teachers pose questions of interest. Questions restricted to classroom | Increased awareness of the statistics question distinction. Students begin to pose their own questions of interest. Questions not restricted to classroom | Students can make the statistics question distinction. Students pose their own questions of interest. Questions seek generalization |

The example below illustrates the differences across the developmental levels for the first component of the process, formulate a question (<http://www.amstat.org/education/gaise/>).

A: How long are the words on this page?

B: Are the words in a chapter of a fifth grade book longer than the words in a chapter of a third grade book?

C: Do fifth grade books use longer words than third grade books?

Cobb (1992) described the importance of question posing in the data production process in statistical thinking. His report in *Heeding the Call for Change* emphasized the need to recognize that it is difficult and time-consuming to formulate problems and to get data of good quality that really deal with the right questions. Students have trouble with this until they go through this experience themselves. Understanding this key role of formulating questions in a way that sets up the entire statistical process is often ignored in classrooms, where teachers and texts start with a set of data and direct students to do something: find the mean, make a plot. MacGillivray (in review) comments that an over-emphasis in school syllabi on answering questions rather than posing them and making decisions based only on data displays produces an approach based on absoluteness of data that stifles the development of statistical thinking. Table 2 shows the development across the three levels of the second component of the process: Collect data.

Table 2 (GAISE)

| Process Component | Level A | Level B | Level C |
|---------------------|--|---|---|
| Collect Data | Do not yet <i>design for differences</i> Census of classroom Simple experiment | Beginning awareness of <i>design for differences</i> Sample surveys Begin to use random selection Comparative experiment Begin to use random allocation | Students make <i>designs for differences</i> Sampling designs with random selection Experimental designs with randomization |

A key element in this process component is recognizing the role of randomness and why it is necessary to have a random sample. Students should understand how bias can occur in the sampling process and start asking critical questions about how samples are chosen. Design flaws can undermine a research project. This was clearly demonstrated in an analysis of articles in peer-reviewed journals on the effects on student achievement of the use of handheld graphing technology in secondary mathematics. The analysis found in many cases the evidence did not support the conclusions because of statistical flaws in the design, at least as described in the papers (Burrill et al, 2002). These flaws included non-random samples, choosing the wrong unit of analysis (mixing students, teachers, schools), and overlooking sources of possible bias. In this process component, students should clearly learn the consequences of poorly designed investigations.

Table 3 describes the GAISE levels related to the third process component, analyze data.
 Table 3 (GAISE)

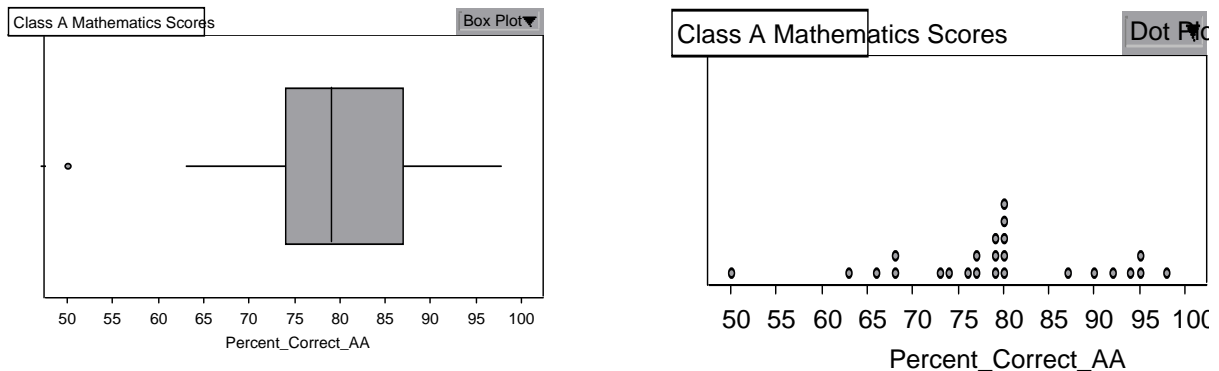
| Process Component | Level A | Level B | Level C |
|---------------------|--|---|--|
| Analyze Data | <p>Use particular properties of <i>distributions</i> in context of specific example Display variability within a group</p> <p>Compare individual to individual Compare individual to group</p> | <p>Learn to <i>use</i> particular properties of <i>distributions</i> as tools of analysis Quantify variability within a group</p> <p>Compare group to group in displays</p> <p>Acknowledge sampling error Some quantification of association Simple models for association</p> | <p>Understand and <i>use</i> <i>distributions</i> in analysis as a global concept Measure variability within a group Measure variability between groups Compare group to group using displays and measures of variability Describe and quantify sampling error Quantification of association Fitting of Models for association</p> |

New research and classroom experiences suggest the need to drastically rethink how this component is presented and developed in the K-12 curriculum. One approach with potentially significant implications for the curriculum is an emerging perspective that, rather than initially beginning with definitions and instruction on routine procedures, central statistical concepts such as measures of center or certain graphical representations might be introduced in ways that promote the development of understanding (Garfield & Ben-Zvi, 2004). Evidence from the research suggests that student understanding of these central concepts is not well grounded. For example, several studies indicate that students have trouble moving from thinking about data as individual points to thinking about the distribution as a whole. Shaughnessy (in press), for example, suggests that students often talk about particular values in the data, without any reference to the holistic characteristics of a distribution, like spread, center, or shape.

Understanding centers is problematic. Early research by Mokros and Russell (1995) indicated that students have problems working with mean as a measure of center in moving from notions of mean as “typical” or “equal shares” to the concept of balance point of a distribution. Cobb, McClain, and Gravemeijer (2003) observed that the U.S. eighth graders in a teaching experiment considered the median only as a cut point in the data and not as a measure of center. A similar observation was made in Dutch and German teaching experiments. As students begin to learn about summary measures for a distribution, Cobb and colleagues claim it may be sufficient for students to consider the median purely as a cut point identifying values as above or below the median. They suggest that additional instruction is necessary to foster understanding of the median as a measure of center and thus as a characteristic of the group. In addition, students tend to conceive of distributions as comprising three parts, rather than four: 1) the majority in the middle (which usually includes more than 50% of the cases), 2) low values, and 3) high values (Bakker & Gravemeijer, 2004; Konold et al., 2002). This suggests two problems for curriculum design: first, the introduction of box plots (which require thinking of data as separated into quartiles) is contrary to the way students think of distributions, and second, students do not think about spread as center plus or minus deviation.

In their Roundtable paper, Bakker, Biehler, and Konold describe research on promising strategies for helping students come to understand measures of center and for interpreting box plots, recommending that early instruction in statistics focus primarily, if not exclusively, on plots in which individual cases are visible. When aggregate plots are introduced, they recommend that the plots initially be accompanied with representations that still allow students to see individual cases, for example, where box plots are overlaid on top of stacked dot plots (Figure 1).

Figure 1



Based on his experience, Harradine (in review), agreeing with Bakker and colleagues, suggests that problems are traditionally posed in ways that require students to read “within, between, and beyond” the data. That is, the student has to make comparisons between two sample data sets and then hypothesize about what that may mean about the population from which the data were drawn. Harradine claims this is too much to ask, initially, for many students. He argues that prior to teaching standard statistical tools and procedures, students should be taught the art of “distribution division” where distributions are sliced into chunks and each chunking is considered to see what information that particular slicing configuration conveys. He also argues that the application of the skills of comparing and contrasting and forming arguments that support a conclusion or conjecture should be taught prior to teaching standard statistical tools. Shaughnessy (in press) is concerned that there has been an overemphasis on centers as the only important concept in statistics, and the important role of variability has been neglected. This is clearly evident in the United States from looking through texts, most of which do not consider any measure of variability, other than a cursory mention of range.

The fourth component in the GAISE framework, interpreting results, is described in Table 4.

| Process Component | Level A | Level B | Level C |
|--------------------------|---|--|--|
| Interpret Results | <p>Do not <i>look beyond the data</i></p> <p>No generalization beyond the classroom</p> <p>Note difference between two individuals with different conditions</p> <p>Observe association in displays</p> | <p>Acknowledge that <i>looking beyond the data</i> is feasible</p> <p>Acknowledge that a sample may or may not be representative of larger population</p> <p>Note difference between two groups with different conditions</p> <p>Aware of distinction between observational study and experiment</p> <p>Note differences in strength of association</p> <p>Basic interpretation of models for association</p> <p>Aware of the distinction between “association” and “cause and effect”</p> | <p>Are able to <i>look beyond the data</i> in some contexts</p> <p>Generalize from sample to population</p> <p>Aware of the effect of randomization on the results of experiments</p> <p>Understand the difference between observational studies and experiments</p> <p>Interpret measures of strength of association</p> <p>Interpret models for association</p> <p>Distinguishes between conclusions from association studies and experiments.</p> |

Note that as students progress through the levels, they should gain a deeper understanding of the basic differences in how observational studies, sample surveys, and experiments are conducted and what types of conclusions can be made from each, recognizing the importance of careful design in each case. Making sense of the analysis, finding connections, and drawing conclusions- in other words, using the

information from the first three components was clearly flagged by those at the Roundtable as problematic. After examining assessment results, Ridgway, McCusker and Nicholson found that top students aged 9 to 13 years were unable to interpret data and apply their statistical ideas in practical situations. Watson and Callingham (in review) found that most students remained at a level characterized by appropriate, but unquestioning, engagement with context, and straightforward application of statistical skills associated with the calculation of simple probabilities and means and graph reading. They argue that this finding suggests that more opportunities need to be created for students to question critically statistical claims from real-world contexts to develop the analytical habits of mind needed to respond critically to quantitative claims. At the collegiate level, Matis, Riley, and Matis observed that the curriculum of many introductory courses is frequently centered solely on the theoretical underpinnings of the subject, often illustrated with simple mathematical exercises that have no practical application or with projects that do not actively involve the students. As a consequence, they found students unable to transfer their knowledge to real contexts.

In summary, the field has begun to identify specific issues that should be considered by those developing and implementing a statistics curriculum. These include:

- begin with a coherent framework that makes clear what the important ideas are and how statistical thinking differs from mathematical thinking;
- pay attention to these key ideas in the framework as materials are developed, for example, helping students learn to ask questions and consider the importance of careful designs for collecting data;
- structure the curriculum in a way that key ideas such as measures of center and spread are scaffolded over the grades with experiences that develop a more complete understanding than currently seems to be the situation;
- provide opportunities in the curriculum for students to exercise their own statistical thinking, applying the concepts they have been studying;
- create frequent situations, appropriate for the grade level and level of development of statistical understanding, where students have to be critical consumers of statistics.

Efforts are being made to address these recommendations but usually on a small scale with a narrow focus. Researchers are designing interventions and monitoring them as was clearly evident at the Roundtable. Suggestions can be found in some of the papers published in journals such as the *Statistics Education Research Journal (SERJ)*, published by the International Association of Statistical Education and ASA's *Journal of Statistics Education*. Others will be in the yearbook on *Thinking and Reasoning with Data and Chance*. A challenge for the community is to link these to the design of curriculum and development of instructional materials. Only then, will we begin to achieve the goal of a literate citizenry able to reason about and with data while taking into account uncertainty.

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