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Universiti Teknologi Malaysia
Reform, Revolution and Paradigm Shifts in Mathematics Education
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New Possibilities in Analysis using the ClassPad300Plus

Ludwig Paditz, University of Applied Sciences Dresden (FH), Germany

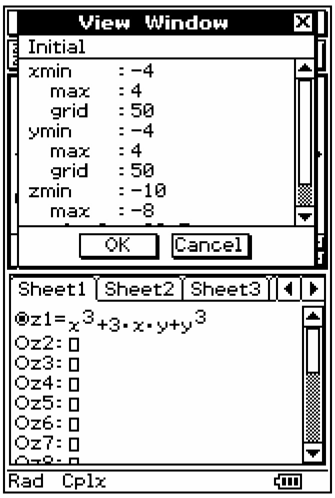
Abstract:

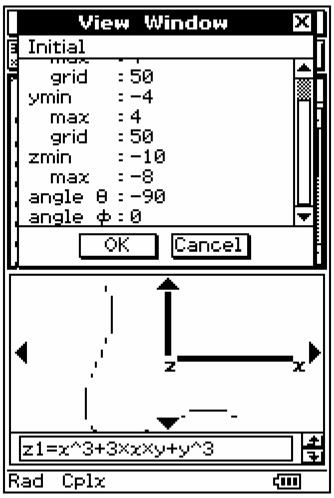
Let us consider the ClassPad300Plus (with the operating system OS 02.20.3010) and discuss on some new exercises in analysis, e.g. implicit plot of the curves f(x,y)=c in the 3D-graphics-window, derivatives of trigonometric functions in degree-mode, root- and logarithmic-functions in real- and complex-mode respectively, ... to see the possibilities using the new tool in the learning process of our students. This lecture continues the other papers [5], [6], [7].
By the help of several examples the interactive work with the ClassPad300Plus is considered. The student can solve difficult exercises of practical applications step by step using the symbolic calculation of the calculator. Sometimes several fields of mathematics are combined to solve a problem, e.g. the 3D-graphics with a small z-range to get in advance a fast imagination on the 2D-graph of the implicit given curves f(x,y)=c.

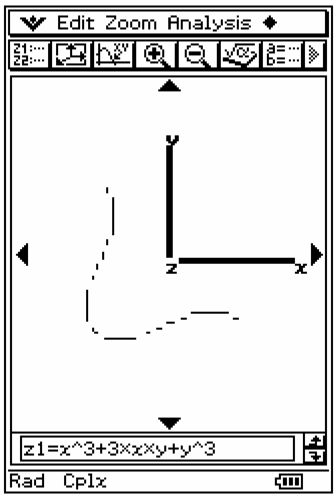
1. Example on an implicit plot:

Consider the 2D-curves **f(x,y) = x^3 + 3*x*y + y^3 = const.** in the x-y-plane and in this connection the 3D-function **z = f(x,y) = x^3 + 3*x*y + y^3**. The aim of this exercise is to get a good imagination on the kind of curves and on the design of the 3D-function respectively. Finally we will compute the local extreme value of the 3D-function. Already in [7] we considered 3D-graphics of 3D-functions.

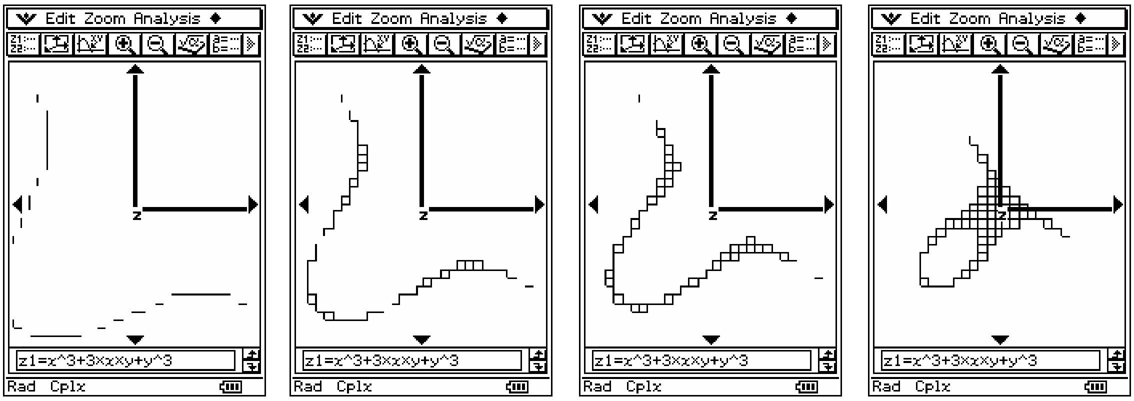
Today we start with the 2D-curves **f(x,y) = x^3 + 3*x*y + y^3 = const.** and consider in the 3D-menu of the CP300Plus the 3D-function **z = f(x,y) = x^3 + 3*x*y + y^3** in the small z-range **c-ε < z < c+ε** , where **c** is the given **const.** and ε is a small positive tolerance.



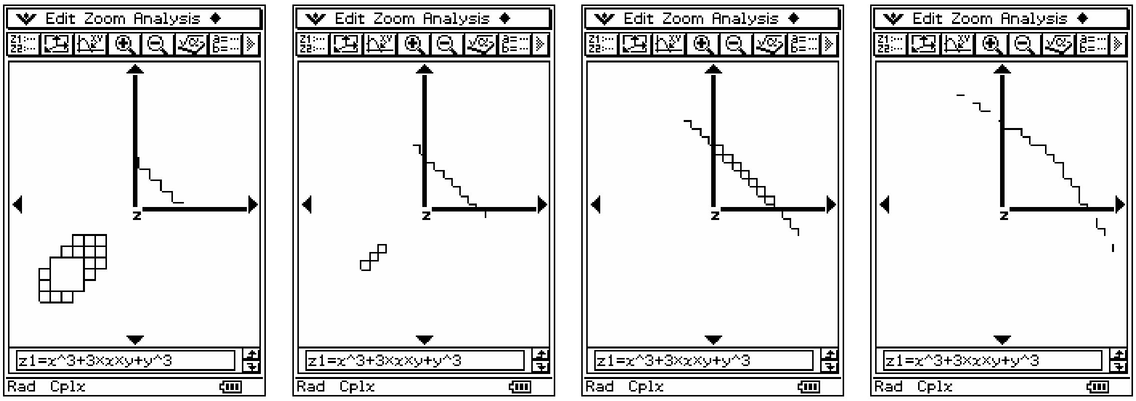




with 4 steps of zoom-in we get the following pictures of the considered 2D-curves:

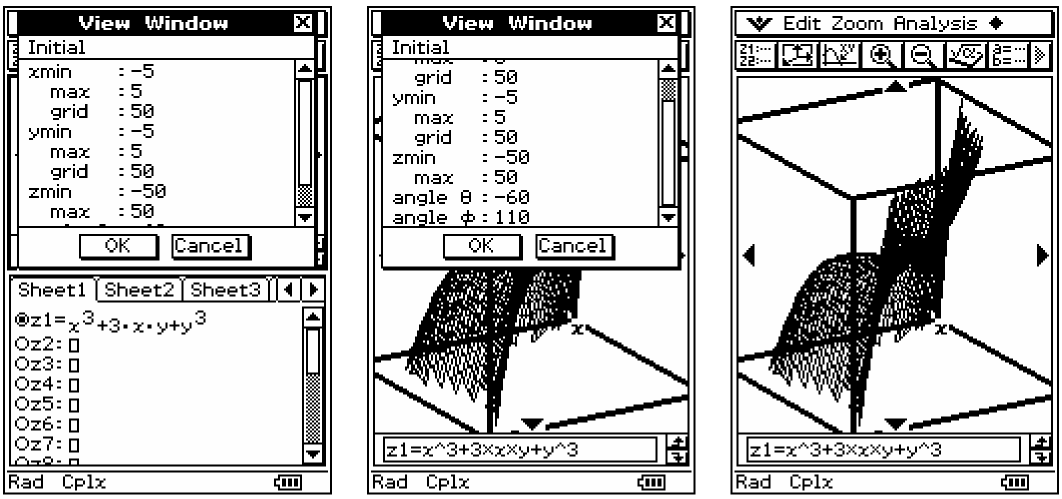


c=-9, ε =1 c=-4, ε =1 c=-2, ε =1 c=0, ε =0.5



c=0.6, ε =0.3 c=1.2, ε =0.4 c=2, ε =1 c=5, ε =1

Here only for a first information we show the 3D-graphics of the surface, which we have scanned in several horizontal planes above:

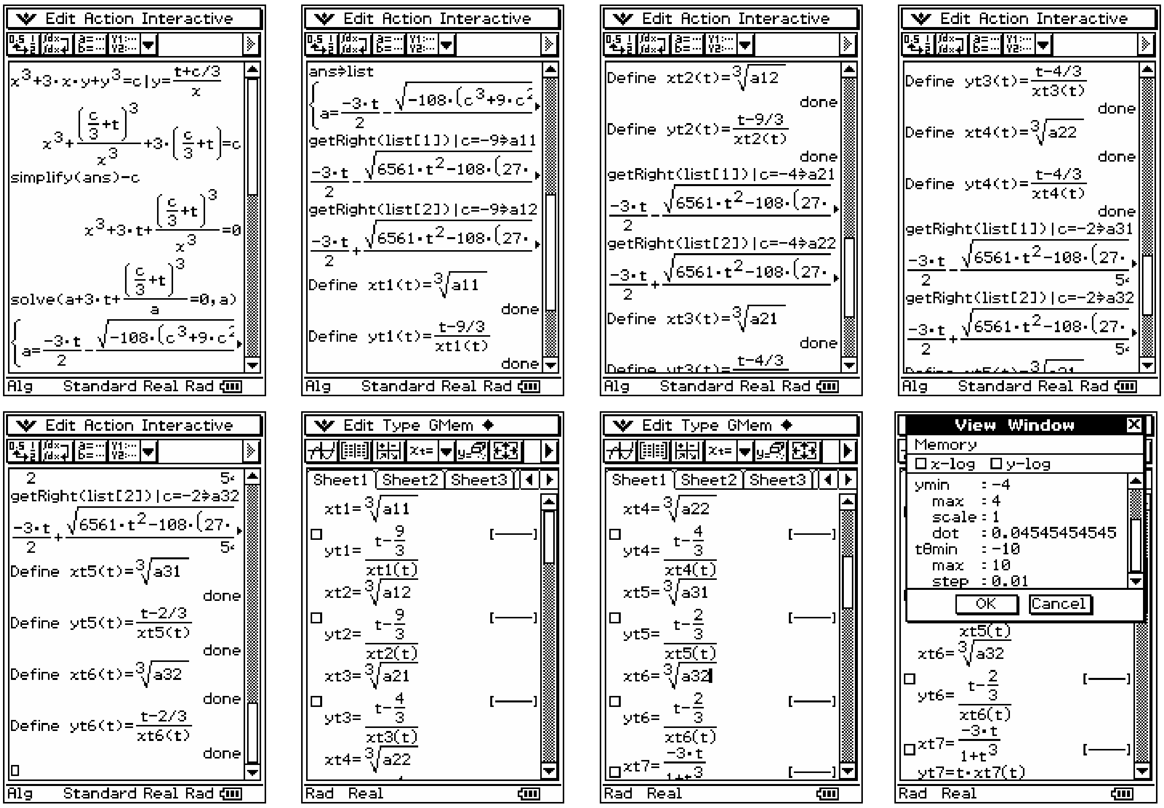


Now we try to draw the 2D-curves in the 2D-graphics-menu of the ClassPad300Plus:

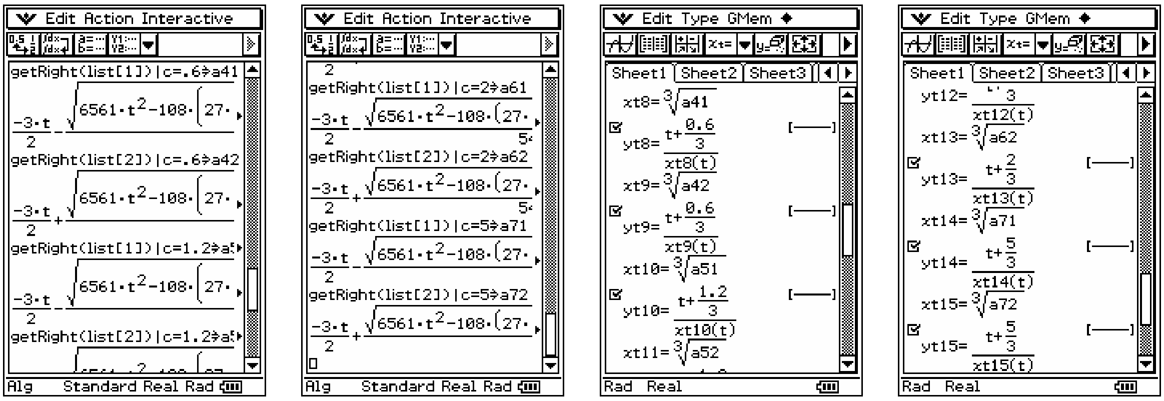
f(x,y) = x^3 + 3*x*y + y^3 = c with **c** $\in$ **{-9, -4, -2, 0, 0.6, 1.2, 2, 5}**

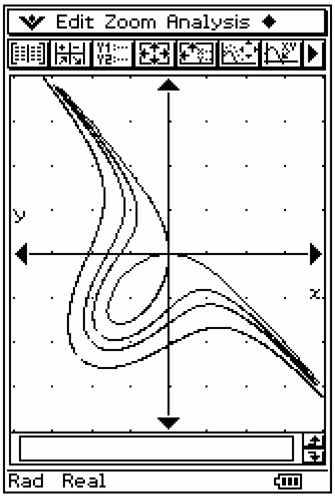
Let us start with **c=0**:
Input **y(t) = t*x(t)** in the equation **x^3 + 3*x*y + y^3 = 0**

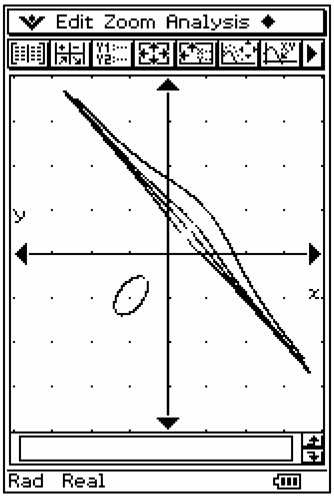
Then we get **x(t)=-3t/(1+t^3)** and can draw this parametric representation in the 2D-window. In the general case the input **y(t) = (t+c/3)/x(t)** is helpful to get a quadratic equation on **a=x^3**. Then in the main-menu we find the parametric representations for **c = -9, -4, -2** :



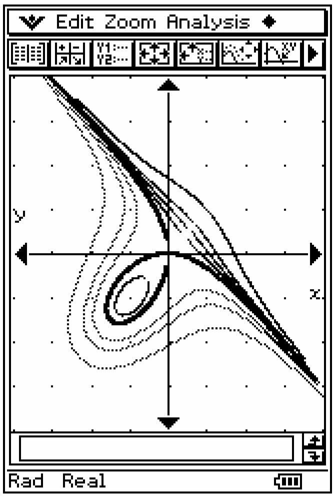
Now the same procedure for **c = 0.6, 1.2, 2, 5**



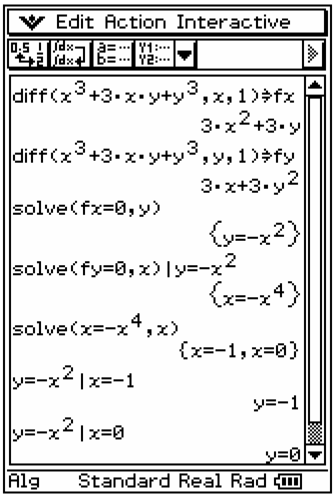


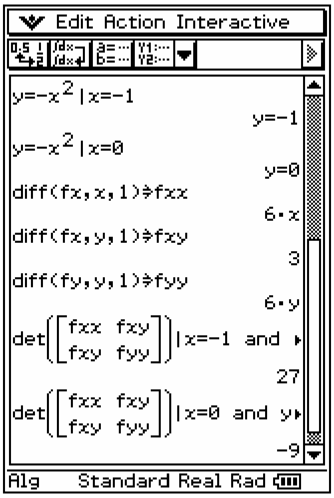


together:



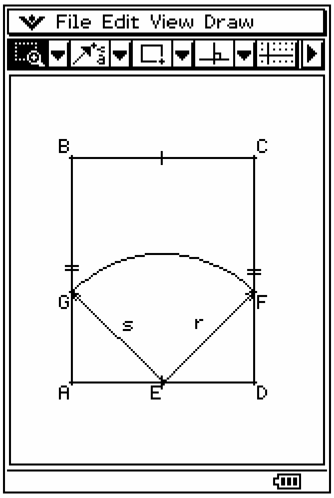
Now we compute the local extreme values of the surface and get for (x,y)=(-1,-1) a local maximum and for (x,y)=(0,0) a saddle point:





2. Example on the derivative of trigonometric functions in degree-mode:

Consider a rectangle ABCD and an arc FG with unknown radius r = s > 0, cp. left picture.



Let be AB = a > 0 and BC = b = a/2 respectively.
Denote the angle GEF with α, where E is the middle of AD.

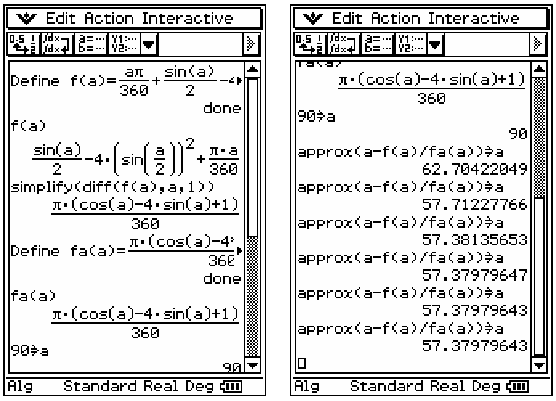
Suppose that the area of the convex figure ADFG equals to the concave figure BCFG.

Find the radius r and angle α respectively, which solve the problem! Use the Newton-iteration to get α.

Solution (α in degree):
$\alpha / 360 = S / (\pi * r^2)$, where S is the sector-area of EFG.
Now the area of the convex figure ADFG is
$S + \Delta DEF + \Delta AEG = a*b/2 = (\pi/360)*\alpha*r^2 + (b/2)*r*\cos(\alpha/2)$

Now with b/2 = r*sin(α/2) and a = 2b we get
4*r^2*(sin(α/2))^2 = (π/360)*α*r^2 + r^2*(sin(α/2))*cos(α/2) and finally

f(α) := (π/360)*α + 0.5*sin(α) – 4*(sin(α/2))^2 = 0, f′(α)=(π/360)*(1+cos(α)-4*sin(α))

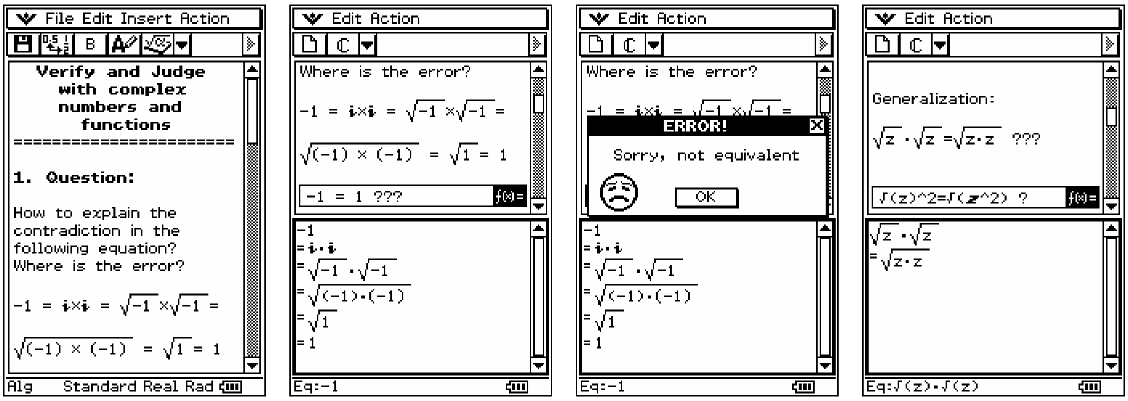


Thus **α = 57,4** [degree] and **r = b/(2* sin(α/2))**.

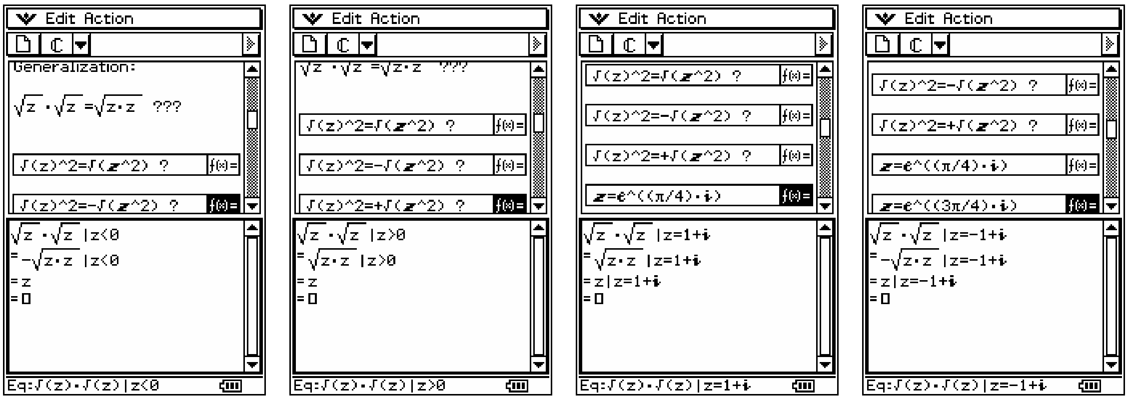
Remark:
In degree-mode the 1st derivative of **sin(α)** is not **cos(α)** but **(π /180)*cos(α)** !
The ClassPad300 computes in the degree-mode with this factor **(π /180)** too.
Sometimes our students forget this factor, because they think in every case the derivative of **sin(α)** is **cos(α)**. However this derivative is only true in radian-mode but not in degree-mode.

3. Examples on complex roots and logarithms:

Let us start an eActivity with the following contradiction:

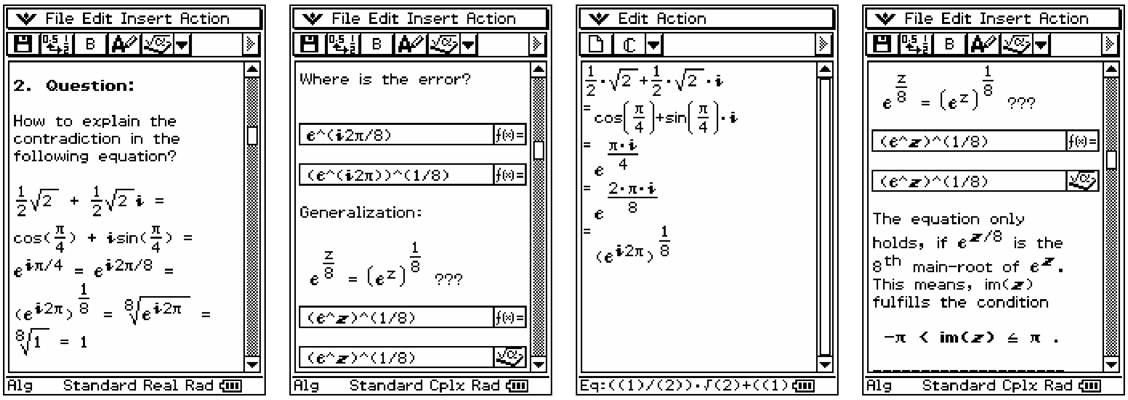


The following equations are true:

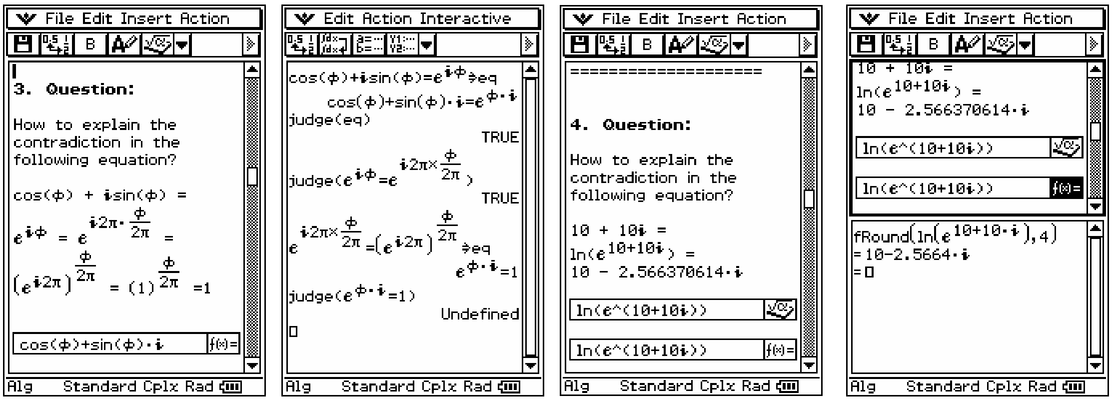


The equation holds, if and only if **re(z)>0 or z=i** , because the root-function gives in every case the complex main-root.

Now consider the following question, which generalize the above discussed problem:

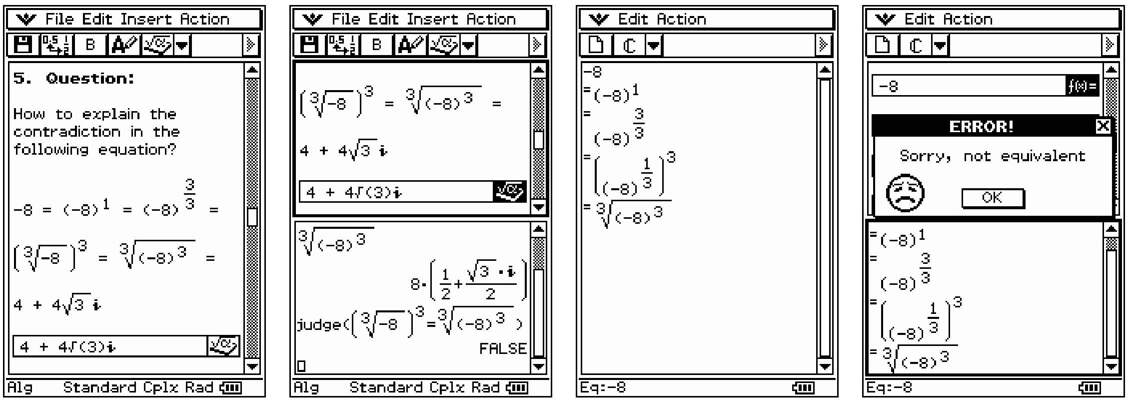


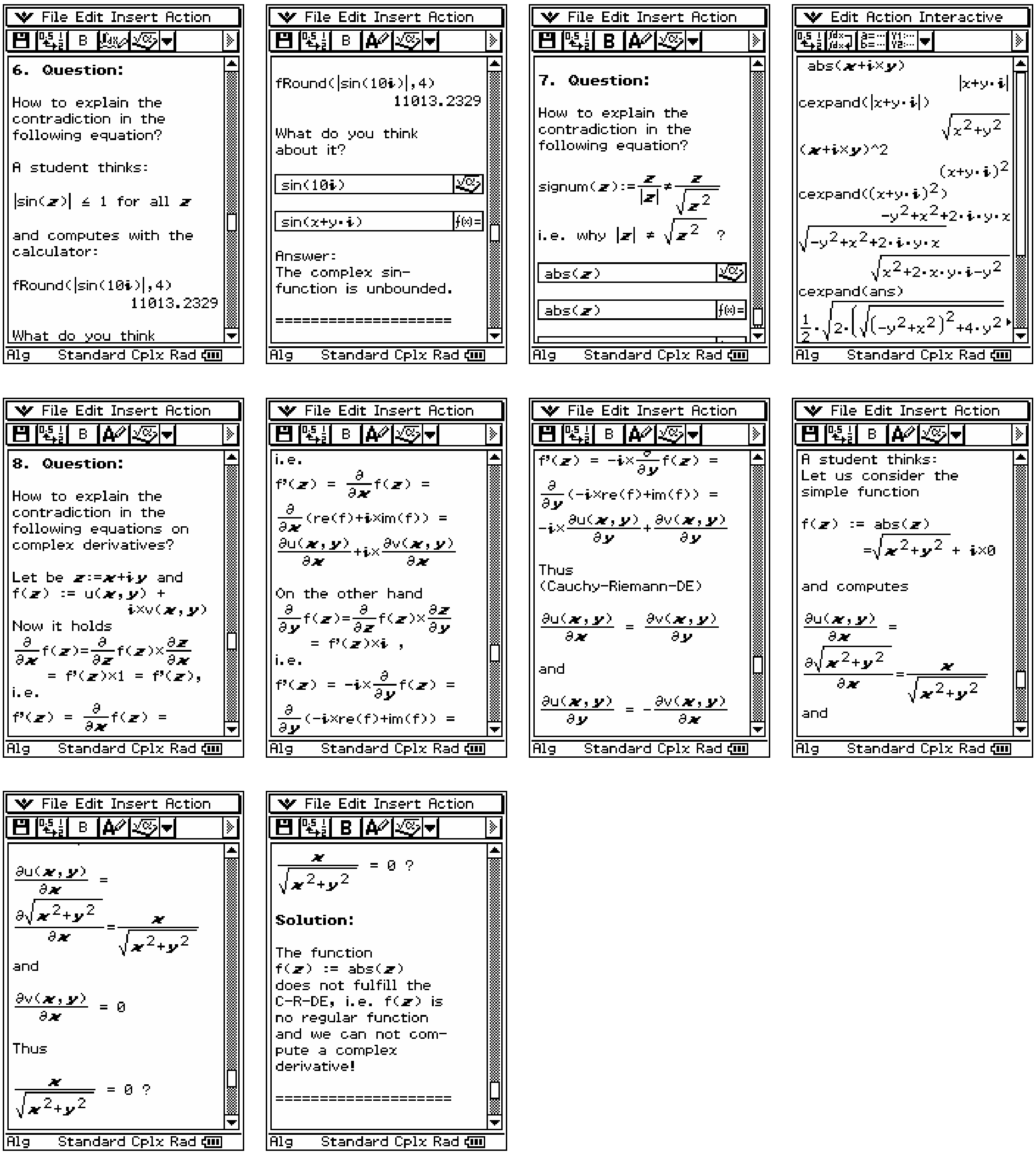
Now the following questions arises:



The discussion with our students serves the better understanding on the main-values of roots or logarithms. By the help of ClassPad300 we can check several situations in this field.

More such questions for discussion with our students:





During the lecture the virtual keyboard with the 2D-input-windows and the possibility of Drag&Drop are used to show how convenient it is to work on the touch-screen.

Contact: paditz@informatik.htw-dresden.de

References:

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[8] **eActivity** (german version: Tschacher, Lang, Liebezeit, engl. translation: Paditz):
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