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THE USE OF METACOGNITIVE PROCESS IN LEARNING MATHEMATICS

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Abstract This article was developed to investigate the process of solving mathematical problems in terms of definable metacognitive behaviours. A case study was used to identify the understanding and to determine the type and pattern of metacognitive behaviours and the associated achievement in mathematical problem-solving. Four Form Four students are used as samples in tackling the mathematical problem using De Corte`s model (2003). At the same time, problems faced by the students when showing the metacognitive behaviours during the process of solving problem are also identified. Data has been collected through verbatim protocol while students undergoing the thinking aloud process in solving three mathematical problem. Methods such as retrospective interview, observation, reviewing of written answers of students were used as the supporting data in this study. Verbatim protocol has been analyzed and coded by using behaviours taxonomy adapted from Foong`s study ( 1993 ) in every phase of De Corte`s model. The research revealed the existence of six types of metacognitive behaviours namely suggest a plan, assess difficulty, review progress, recognize error, new development and self-questioning. The existence of these six types of metacognitive behaviours has been found interrelated within the five phases of De Corte`s model. The research has shown that there is a strong relationship between the types of metacognitive behaviours and the performance of students in the process of problem solving.

INTRODUCTION

In the Curriculum and Evaluation Standards for School Mathematics, NCTM (2000) has placed problem- solving as a major vision in mathematics education besides reasoning, communicating and connecting. Problem-solving is a complex process which involves several cognitive operations such as collecting and selecting information, heuristic strategy and metacognition (Garofalo & Lester, 1985; Schoenfeld, 1994 and De Corte, 1995). Recent research in mathematics education indicates that more and more attention has been given to metacognition. Metacognition refers to the idea of examining one's own knowledge and thoughts. It also relates to active monitoring and regulating of thought process. The term metacognitive behavior was used to describe statements made by students about the problem or made about the problem-solving process. This research article described the types of metacognitive behaviours which exists and interrelates with student's achievement in the process of problem-solving.

OBJECTIVE

The aim of this research is to investigate the metacognitive behaviour in every phase of De Corte`s model ( 2003 ) while students are solving mathematical problems. In this research, data and information had been collected and analyzed in order to identify the types and pattern of metacognitive behaviours which are involved in the 5 phases of De Corte`s model, together with the relationship between metacognitive behaviours and the achievement during the problem-solving process. Simultaneously, problems faced by students during the mathematical problem-solving process are also identified.

METHODOLOGY

This research involved four Form Four students from a secondary school in Johor Bahru, Malaysia. It was carried out by using qualitative approach in order to understand the metacognitive behaviours during the problem-solving process. The protocols in getting the verbatim transcription via the thinking aloud method which was recorded in audio was carried out. Data collection also consisted of observation during thinking aloud process, retrospective interview and reviewing the students` written answers which were used as supporting data. The four samples are classmate of the same secondary school which was in operation since July 2001. All the samples had scored A in mathematics during PMR(Malaysian Standardize Examination for lower secondary students) in year 2003. The samples have similar traits that is having an interest in mathematics and are capable to voice their opinions. These are important traits as the samples were required to say out all activities in their mind during the thinking aloud process.

The samples were trained to carry out the thinking aloud method for two weeks under close guidance. After that, the samples were requested to solve three non-routine mathematical problems concerning daily life task. Each sample was required to solve three mathematical problems by using the thinking aloud method individually. The problems were distributed to each sample on a typed sheet of paper. Each sample was instructed to state verbally on anything that came to his/her mind when they

solved the problems. The whole session was recorded in audio in order to capture every word that was stated by the samples. No time limit was imposed and the samples were asked to draft out the solution, steps or diagrams on a sheet of paper. Observation was recorded during the whole experiment. Types of metacognitive behaviours were modified from the behaviour taxonomy in Foong`s study (1993).

Retrospective interview was carried out right after the thinking aloud process. Attention had been given to the parts that were unclear to the researcher in the thinking aloud process, observation and drafting answers . The retrospective interview was focused on the types of metacognitive behaviours and problems faced by samples when trying to express the metacognitive behaviour during solving the problems.

DATA ANALYSIS

The thinking aloud protocols during the problem-solving process were collected from four samples. Each problem-solving session was recorded in its entirety and the audio recorded protocols were transcribed verbatim. Twelve sets of protocols were collected from four samples through thinking aloud method as the main data, and was supported by three other data, that are observation, retrospective interview and samples` written answers. Each transcribed protocol was divided into segments of behaviours. Each segment was then classified and encoded according to Schoenfeld`s method (1985) and also using modified taxonomy of problem-solving behaviour from Foong`s study (1993).

RESULTS

The protocol analysis in the problem-solving process had outline five categories of behaviours, that are Problem-Orientation Heuristic (P), Problem-Solution Heuristic (H), Domain-Specific Knowledge (K), Affective Behaviour (A) and Metacognitive Behaviour (M1 – M6). The five Phases in De Corte`s model (2003) are also recognized for each transcribed protocol. From the protocols, six types of metacognitive behaviours, that are Suggest A Plan (M1), Assess Task Facility (M2), Review Progress (M3), Recognise Error (M4), Recognise New Development (M5) and Self-questioning (M6) had involved in five phases in De Corte`s model, that are Phase I (build a mental representation of the problem), Phase II (decide how to solve the problem), Phase III (execute the necessary calculations), Phase IV (interpret the outcome and formulate an answer) and Phase V (evaluate the solution).

Table 1shows the frequency of metacognitive behaviours in each phase for each sample during the problem-solving process. It shows that all six types of metacognitive behaviours were present for all problems with a certain frequency. In summary, M4 and M3 behaviour occurred most that was 24 times for each, followed by M2 that was 14 times, M1 for 13 times, M5 for 12 times and the least was M6 which only occurred 9 times. The table also indicates that the highest frequency of metacognitive behaviours were M3 and M4. Meanwhile Phase IV marked the highest presence for metacognitive behaviours, that is 46 times.

Table 2 summarized the frequency of metacognitive behaviours with problems, samples and phases in the problem-solving process. In the problem-solving achievement, Pan (not her real name) with the highest frequency in metacognitive behaviours scored the highest achievement. She scored all three problems distinctively, Don solved two problems and a part of another, Fad could only solve one while Vim could not solve any problem.

Types / Phase	M1	M2	M3	M4	M5	M6	Total
PI	1	6	2	0	2	1	12
PII	11	6	2	2	2	1	24
PIII	0	0	5	2	0	1	8
PIV	1	1	10	20	8	6	46
PV	0	1	5	0	0	0	6
Total	13	14	2 4	24	12	9	

Table 1 : Frequency Of Types Of Metacognitive Behaviours For Each Phase

Level of Frequency	Metacognitive Behaviours	Problem	Sample	Phase
high	M3, M4	first	Pan	PIV
	M2			PII
	M1	second	Don	PI
	M5		Vim	PIII
low	M6	third	Fad	PV

Table 2 : Frequency Of Metacognitive Behaviours With Associated To Problems, Samples and Phases

DISCUSSIONS AND IMPLICATIONS

The problem-solving model introduced by De Corte (2003) can help and guide students to use metacognitive strategy in their problem-solving process. Six types of metacognitive behaviours were traced in five phases De Corte`s model with their own pattern and frequency. This can be shown as Figure 3 .From Figure 3, behaviour assess task facility (M2) and recognize new development (M5) in the phase of building a mental representation of the problem were often occurred. It also showed that M2 and M5 were important in this phase. Therefore, students were able to build up a clear image towards the task facility and then to plan for the next step.

While reading the question and observing the requirements of the problem, students were also be able to find a good and useful mental representation to create an idea or development towards the problem. Therefore, the students must be capable of connecting concepts with mathematical thinking such as justification, abstraction, generalization and interrelation (Tall, 1997).

In Phase II, which decides how to solve the problem involved the M1 (suggest a plan) and M2 (assess task facility) behaviours. Here, students must be capable to connect the requirements of problem while strategically making plans to solve the problem. In the phase that executed the necessary calculations (Phase III), metacognitive behaviours for reviewing progress (M3) and recognize error (M4) are important. Students with high self-regulation would continuously keeps questioning themselves. It could also help the students to check on what had been done during the calculation and traced out the mistakes that had been made. Students who were able to trace out errors will refer to Phase I and II in order to find out some important information and to make some adjustments of action

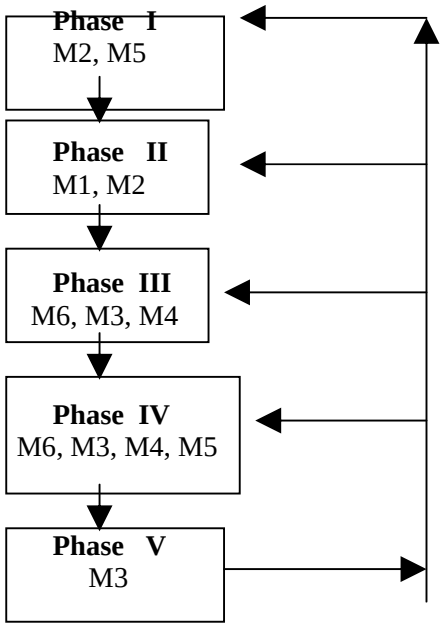

Table 3 : Flowchart Showing The Relationship Of Metacognitive Behaviours (high frequency) With Phase

Students that are capable in self-regulation would be encouraging themselves to recheck and to trace out the errors when interpreting the outcome during Phase IV. This always can help the students to come out with a new idea. With this, students can make sense in their problem-solving process. If a student could not get anything after reviewing progress, the student might refer back to Phase I, II and III to do some regulations action.

In Phase V (evaluate the solution), the metacognitive behaviour M3 (review progress) occurred frequently. This was to ensure the calculations was done in a proper way without leaving out important information and answers. Students would reflect to other phases if they had some hesitations in their solution. If the student could get through Phase V effectively, she/he can be able to answer mathematical question successfully.

There are significant relationship between the types of metacognitive behaviours and the achievements in solving problem. The research indicates thare are differences in metacognitives behaviors between high acievement students and average students in their problem-solving process. For high achievemet students, they could make use of the phases (PI – PV) while solving the problems. Besides, they were able to apply the types of metacognitive behaviours in a useful and effective way for each phase. The frequency of metacognitive behaviours would be higher and the problem-solving process became systematic and under control by their high self-regulation traits compared to lower achievement students.

A low degree in self-regulation and not be able to develop the comprehension monitoring could be blocking students to use metacognitive behaviours in problem- solving process. Weakness of students to interpret information would affect his/her in comprehension monitoring, and it could hamper the students from making logical decisions. Despite of that, negative affective traits and lack of proper understanding in the nature of mathematics could also affect the presentation of metacognitive behaviours.

CONCLUSION

This research has successfully traced out the types and pattern of metacognitive behaviours in every phases in the De Corte`s model. These types of metacognitive behaviours are suggest a plan, assess task facility, review progress, recognize error, recognize new development and self-questioning. Each behaviour varies in its own frequency in certain phases. Besides that, this research has shown that how the metacognitive behaviours had affected the performance of problem-solving. Each type of metacognitive behaviours played its own role to help the students to solve the problems. This research has an important implication in the process of learning and teaching of mathematics in class especially in problem-solving process.

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