Developing mathematical thinking with the assistance of manipulatives

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Abstract: *The use of manipulatives as part of mathematics lessons has long been advocated as part of a comprehensive mathematics learning experience. Recent developments such as virtual manipulatives, along with research have caused some to question the role that manipulatives play in learning mathematics. In this paper the authors re-examine the use of manipulatives within the constructivist paradigm. Pattern Blocks are used to illustrate how mathematical thinking may be developed with the aid of manipulatives.*

Introduction

For many years teachers of mathematics, particularly at the primary school level, have espoused the virtues of 'hands on' learning. Often the term concrete materials was used to describe mathematics materials such as Base Ten Blocks, Unifix materials or Pattern Blocks. Given the development of virtual manipulative the definition offered by Perry and Howard (1997) has been adopted for this paper. They defined manipulatives as "all materials, both inside and beyond the mathematics classroom, which can be experienced through senses of sight, touch and/or sound" (p. 26).

Manipulatives come in various forms:

- Unstructured
- Structured and
- Virtual. (See http://matti.usu.edu/nlvm/nav/vlibrary.html)

For the most part in this paper we will refer to the structured aid, Pattern Blocks. Unstructured aids include items such as buttons, pop-sticks and the like. Virtual manipulatives simulate the physical manipulative in the computer environment. The structured manipulative, Pattern Blocks may also be found in the virtual environment.

Justification for the use of manipulatives

The ancient Chinese Proverb;

I hear and I forget
I see and I remember
I do and I understand

is often quoted as a justification for the use of manipulatives. Swan & Sparrow (2004) discussed this spurious argument and proposed a model for using manipulatives that incorporates the role of reflection and discussion, not just simply action. In fact children may look as though they are busy using manipulatives but they are not necessarily learning.

While children may have appeared engaged in the task of 'manipulating blocks' it is not clear what they were learning. In some cases rather than construct appropriate knowledge children were in fact mis-learning or mis-constructing knowledge.

Ball, 1992, p. 17 (as cited in Perry and Howard) noted:

> One of the reasons that we as adults may overstate the power of concrete representations to deliver accurate mathematical messages is that we are 'seeing' concepts that we already understand. That is, we who already have the conventional mathematics understandings can 'see' correct ideas in the material representations. But for children who do not have the same mathematical understandings that we have, other things can reasonably be 'seen'.

As Clements (1999, p. 2) so eloquently sums up "[I]t cannot be assumed that concepts can be 'read off' manipulatives." It cannot be assumed that simply using manipulatives means that children will learn. If children are to construct meaning from the manipulative being used, teachers need to be explicit about the mathematics to be developed from the manipulative. Children need to be given time to gain familiarity with the manipulative so that rather than focus on the manipulative itself they will be focused on the mathematics to be developed. Stein and Bovalino (2001) noted that: "If not used with careful thought, manipulatives can become little more than window dressing, they are nice to look at and play with but superfluous to overall learning" (p. 357).

The use of manipulatives may be justified in a setting where the teacher uses the manipulative as a catalyst to encourage thinking about a topic. In addition this thinking needs to be teased out by student talk in order that teachers may gauge whether appropriate knowledge is being constructed. Clearly observation of children working with manipulatives is a powerful tool for deciding whether learning is taking place. Skemp (1986) described observing children using manipulatives as providing an insight into their thinking when he state "it is as if their thinking [that of children] is out there on the table." In the digital age it is not difficult to photograph children's work and ask them to write about the learning that took place and is taking place.

A brief overview of manipulatives research

The use of manipulatives has been studied over a number of years. The advent of virtual manipulatives has rekindled interest in the role that manipulatives have to play in the learning of mathematics. The following is a brief outline key research findings.

- Students who use manipulatives outperform those who don't (Clements, 1999). Kennedy (1986) noted that: "Although no single study validates the claim that children should use manipulative materials as they learn mathematics, the collective message garnered from many studies is that materials are worthwhile" (p. 7).

- Attitudes toward mathematics improve when concrete materials are used (although there is a caveat provided … teachers are "knowledgeable about their use" (Clements, 1999, p.1).

- Use of manipulatives declines in later years of the primary school (Gilbert and Bush, 1988; Perry and Howard, 1997). It is interesting to note that one of the reasons for this decline in use appears to be a lack of teacher knowledge. This includes a lack of knowledge of how to manage and how to use the manipulative as well as a lack of knowledge of the associated mathematics being developed.

A change in approach

Clements (1999) questioned the whole notion of moving from the "concrete to the abstract". He revisited this commonly accepted approach, especially in the light of computer or virtual manipulatives. He stated:

> …common perspectives on using manipulatives should be reconsidered. Teachers and students should avoid using manipulatives as an end -without careful thought - rather than as a means to that end. A manipulative's physical nature does not carry the meaning of a mathematical idea. Manipulatives alone are not sufficient - they must be used in context of educational tasks to actively engage children's thinking with teacher guidance. (Clements, p.9)

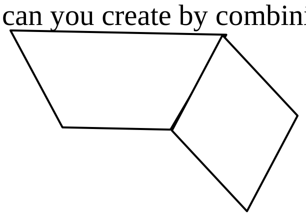
Likewise Stein and Bovalino (2001), who studied teacher's use of manipulatives, indicated that teachers either tended to step children through the use of manipulatives, allowing very little latitude or else left them very much to their own devices. The authors believe that a middle ground needs to be struck whereby the children are given time to explore the manipulative but are encouraged toward a particular goal. In the following section a common manipulative, Pattern Blocks, available in physical and virtual form is used to illustrate how children may be aided to construct knowledge.

Working Mathematically with Manipulatives: Pattern Blocks

Pattern blocks are made up of a yellow hexagon, red trapezium, wide blue rhombus, thin white (tan) rhombus, green triangle and orange square. The side-lengths and angle sizes of the various pieces are all related

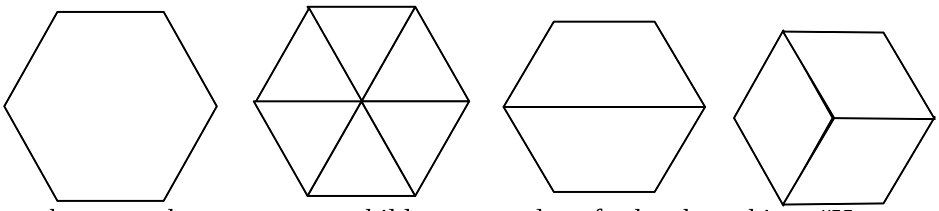
Make a hexagon
It is often the case that children visualise a hexagon as regular. The yellow hexagon that forms part of the Pattern Block manipulatives may contribute, inadvertently to this thinking. Here is an example where knowledge may be misconstructed. Skilful use of the manipulative helps overcome any such misconceptions. Consider the following activity.

How many different hexagons can you create by combining two of the Pattern Block pieces?

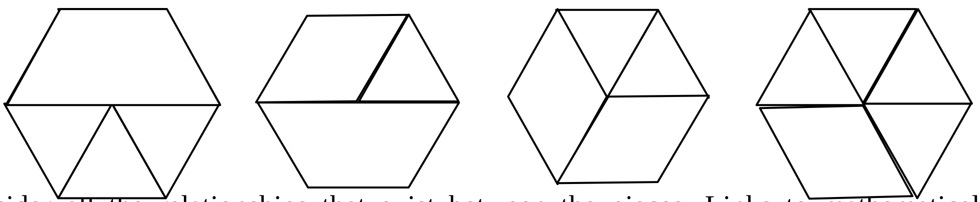


Make a yellow hexagon

This task may be used to introduce fraction ideas. Children are set the task of finding ways to make a yellow hexagon using other pattern block pieces. Children often find the obvious cover ups such as two red trapeziums six green triangles and three blue rhombuses.



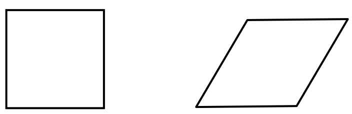
The teacher may then encourage children to explore further by asking, “How many different ways there are to cover up a hexagon?” and “How will you know when you have found them all?”. This will serve to encourage mathematical thinking and enquiry. Children will need to be systematic and record all their solutions.



Consider all the relationships that exist between the pieces. Links to mathematical symbols and fraction ideas may then be made. What if … questions may be asked to encourage further thinking about the relationships between the pieces. For example, what if two hexagons represent one whole?

Area and Perimeter relationships

As described earlier all the Pattern Block pieces are related. Edge lengths are a single unit (2.5 cm or one inch), except the red trapezium which has a base that is two units long. A variety of questions may be asked in order to elict children’s understanding of perimeter and area. An example of such a question designed to raise conflict in the mind of the child involves comparing the orange square and the blue rhombus and indicating that both have the same perimeter by rolling one shape around the other.



Then the erroneous statement that as both shapes have the same perimeter they have the same area – prove it! Children rarely even question this and eagerly attempt to try to prove this. Rarely do children consider the tan rhombus, which also has the same perimeter, but quite clearly a much smaller area. The discussion that such a task generates can be most rich.

Angle

Many children view angle as simply a measurement made with a protractor. Setting a task such as determining all the angle sizes of the Pattern Block pieces without using a protractor can at first seem a little difficult. A careful observation of how children tackle this problem indicates that they start to make use of their background knowledge such as there are 90 degrees in a corner, or 180 degrees in a straight line or 360 degrees in a circle. Using differing starting points children then work out the angles sizes in some pieces and use these determine the angle sizes of the other pieces. The task of representing all the angle sizes from 30 degrees to 360 degrees using individual pieces or combinations of pieces may then be set.

Conclusion

Manipulatives per se do not teach in and by themselves. They may be used to help children to construct knowledge, but only if there is a clear purpose for the activity in the teacher's mind. This is then translated into activities where this purpose is highlighted by the use of pertinent focus questions which allow children to build on their existing knowledge, not by accident but by design.

References

Clements, D. H. (1999). Concrete manipulatives, concrete ideas. *Contemporary Issues in Early Childhood*, *1*(1), 45-60.

Gilbert, R., & Bush, W. (1988). Familiarity, availability, and use of manipulative devices in mathematics at the primary level. *School Science and Mathematics, 88* (6), 459-469.

Kennedy, L. M. (1986). A rationale. *Arithmetic Teacher, 33*(6), 6-7, 32.

National Library of Virtual Manipulatives for Interactive Mathematics. (2003). Retrieved September 6, 2004 from http://matti.usu.edu/nlvm/nav/vlibrary.html

Perry, B., & Howard, P. (1997). Manipulatives in primary mathematics: Implications for learning and teaching. *Australian Primary Mathematics Classroom 2*(2), 25 – 30.

Ross, R., & Kurtz, R. (1993). Making manipulatives work: A strategy for success. *Arithmetic teacher, 40*(5), 254-257.

Skemp, R. (1986). *Twice five plus the wings of a bird*. [Video]. London: BBC.

Stein, M. K., & Bovalino, J. W. (2001). Manipulatives: One piece of the puzzle. *Mathematics Teaching in the Middle School, 6*(6), 356 – 359.