The Mathematics Education into the 21st Century Project
Universiti Teknologi Malaysia
Reform, Revolution and Paradigm Shifts in Mathematics Education
Johor Bahru, Malaysia, Nov 25th – Dec 1st 2005

Conceptests: Active Learning In Calculus

Deborah Hughes Hallett, Maria Robinson, David Lomen

ConcepTests—a powerful tool for improving student learning—were originally developed by Eric Mazur at Harvard to teach introductory physics. They were subsequently adopted in chemistry and biology. ConcepTests are now available for calculus, where they have shown the same impressive results. The pedagogy underlying Conceptests is active learning and peer-instruction. These have proved effective in many contexts, particularly for non-traditional students who do not learn as well in a passive lecture format.

Over the past two decades, many faculty have found ways to improve student learning by making better, more active, use of class time. One of the most effective methods—the use of ConcepTests and peer instruction---was developed by Eric Mazur of Harvard. His pioneering work in physics has been successfully replicated in other departments, and its efficacy is clearly supported by data. Information is available under "Peer Instruction" at http://galileo.harvard.edu.

ConcepTests have now been written for calculus. [1] This is the first time the method has been made widely available for mathematics.[*] Evaluation suggests that ConcepTests are as effective in calculus as they are in physics.

What are ConcepTests and How are They Used?

ConcepTests are questions designed to promote the discussion and learning of mathematical concepts during a calculus class. The questions are usually conceptual, often multiple choice or true/false, with some free response questions as well. They are used as an aid in promoting student involvement in discussing mathematical concepts rather than as a method of testing students for a grade. (Some questions have more than one correct answer.)

Because of the variety of forms of these questions, instructors can use them in several different ways. Three of the possibilities are:

♦ As an introduction to a topic. This works especially well if the topic is closely tied to previous lessons or is something with which most students have some familiarity.

♦ After presentation of a specific topic by the teacher. Here the ConcepTest may be used to see if the students have grasped the concept, or if the topic needs more discussion or examples.

♦ As a review of material that has been thoroughly discussed.

Instructors usually display the ConcepTest using an overhead projector, or distribute a copy to each student. Students are then given short time (one to four minutes, depending on the question) to think about the question and then vote for the answer they think is correct. Providing almost all do not vote for the correct answer, the students are then given a few minutes to discuss the ConcepTest with adjacent students and then are given another chance to vote on the correct answer. Students are also asked to develop reasons to support their answer. The instructor then discusses the correct answer (or answers) and has students present their reasons.

Why do ConcepTests Work?

Calculus instructors often strive to involve students in classroom discussions. However, it is not uncommon for their efforts to be met by silence as students wait for the instructor to answer his or her own questions. ConcepTests work because they remove the barriers to student response. At the outset, instructor and students both know that there will be several minutes to grapple with the question, so there is no pressure on the instructor to answer too quickly. The responses are essentially anonymous, so students can comfortably put forth tentative ideas. If the students are lost or unsure, they have the opportunity to talk things over with their classmates and to revise their first impressions. The give-and-take helps both weak and strong students and, even if it does not generate a correct answer, often generates excellent questions from the students to the teacher.

Regular use of ConcepTests ensures that students first grapple with difficult mathematical ideas in class, where help is available, rather than on homework, when help is not available. Having wrestled with ideas in class, students are much more likely to be able to tackle problems actively on their own.

What Kinds of Classes Benefit from ConcepTests?

---

[*] This paper is based on the introduction to *ConcepTests* which accompanies *Calculus*, by D. Hughes Hallett, A.M. Gleason, et al. (John Wiley).

While originally developed for use in a large classroom, we find ConcepTests equally effective in small or moderate sized classes, where they promote ``active'' or ``discovery'' student learning. The ensuing discussion greatly increases the students' familiarity with the subject and helps them formulate mathematical ideas in their own words. This increases the students' base of knowledge and enthusiasm for mathematics.

Evaluation of ConcepTests

Scott Pilzer describes [2] Eric Mazur's development of ConcepTests and Mazur's evaluation data. Scott Pilzer, who taught with ConcepTests in physics before writing them for calculus, gathered similar evaluation data for mathematics. He found that at the start of the subsequent semester, students taught Calculus I using ConcepTests and peer instruction performed much better on conceptual questions and somewhat better on standard computational problems than those who had been taught standard lectures. See Table 1.

TABLE 1
Comparison

	Conceptual Questions	Computational Problems
With ConcepTests	73%	63%
Standard Lecture	17%	54%

From Scott Pilzer.

These dramatic results mirror the results found in physics: a large increase in conceptual understanding in addition to an increase in standard computational problem solving ability. In his article, Scott Pilzer describes how this enthusiasm translated into more mathematics majors.

We hope that many departments consider teaching with ConcepTests and taking part in developing expertise with a pedagogy that promises to be as productive for mathematics as it has been for physics.

References

1. S. Pilzer, M. Robinson, D. Lomen, D. Flath, D. Hughes Hallett, B. Lahme, J. Morris, W. McCallum, J. Thrash, *ConcepTests,* John Wiley, 2003.
2. Scott Pilzer, *Peer Instruction in Physics and Mathematics,* Primus, **XI(2)** (June 2001), 185-192.

Deborah Hughes Hallett
University of Arizona
Tucson, Az
USA
<dhh@math.arizona.edu>

Maria Robinson
Seattle University
Seattle, WA
USA
<robinsonm@seattleu.edu>

David Lomen
University of Arizona
Tucson, Az
USA
<lomen@math.arizona.edu>

The Mathematics Education into the 21ˢᵗ Century Project
Universiti Teknologi Malaysia
Reform, Revolution and Paradigm Shifts in Mathematics Education
Johor Bahru, Malaysia, Nov 25ᵗʰ – Dec 1ˢᵗ 2005

Appendix A: Examples of Concep Tests

1. Using the graph in Figure 1, arrange in ascending order:

   a) Slope where $x = 0.2$

   b) Slope where $x = 1.5$

   c) Slope where $x = 1.9$

   d) Slope of line connecting the points where
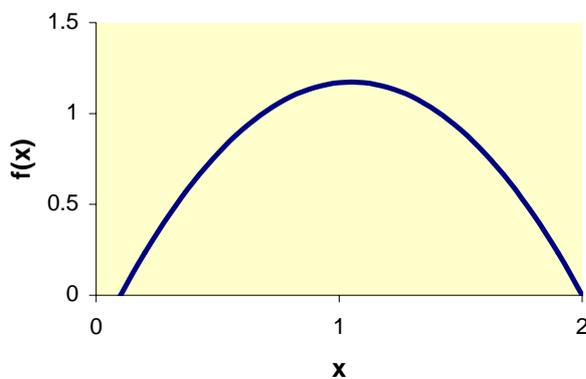 $x = 1.5$ and $x = 1.9$

   e) The number 1

FIGURE 1.*Graph of function*

2. Let $N = f(t)$ be the total number of cans that Sean has consumed by age t in years. Interpret the following in practical terms, paying attention to units.

   a) $f(14) = 400$

   b) $f^{-1}(50) = 6$

   c) $f'(12) = 50$

   d) $(f^{-1})'(450) = 1/70$