

Emphasis on Problem Solving in Mathematics Textbooks from Two Different Reform Movements

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Both the *New Math* reform of the 1960s and the *Standards-based* reform starting in the late 1980s significantly influenced mathematics education within and beyond the frontiers of the United States. The emphasis was not only on new content but also on how this content was to be taught in class. Problem solving, which is the backbone of mathematics education, was portrayed differently in these two reform movements. To frame my discussion on the emphasis on problem solving in these two reform movements, I have chosen the *School Mathematics Project Study* (SMSG) curriculum materials from the 1960s and the fairly recent *Connected Mathematics Project* (CMP) textbooks of grades 6-8, which were funded by the National Science Foundation (NSF) and developed nearly 40 years apart. My comments will be based on what could be inferred from the nature of problems and the emphasis on problem solving in the curriculum materials and also on the overarching philosophical and the psychological perspectives in the reform movements. What are problems and what is problem solving?

Problems and Problem Solving

Problem solving has been used with multiple meanings that range from “working rote exercises” to “doing mathematics as a professional” (Schoenfeld, 1992). To understand what is problem solving, we must first of all understand what is a problem. If the answer to a problem is apparent then it is no longer a problem. Hence, the defining feature of a problem situation is that there must be some blockage on the part of the potential problem solver. Krulik and Rudnik (1980) defined a problem as a situation that requires resolution and for which the individual sees no apparent or obvious means or path to obtaining the solution. It should be added that the solver should be motivated to reach the solution. What is a problem for one person may not necessarily be a problem for another person. Polya (1957) differentiated between routine and non-routine problems. While routine problems are mere exercises that can be solved by some rules or algorithms, non-routine problems are more challenging, and they require some degree of creativity and originality from the solver. Polya added that it is only through the judicious use of non-routine problems that students can develop problem solving ability. So, problem solving is not just solving a problem. It is the process by which students experience the power and usefulness of mathematics in the world around them and it also a method of inquiry and application (NCTM, 1989). Thus, problem solving is a complex process which Polya claimed proceeds through his much publicized four phases: *understanding the problem*, *devising a plan*, *carrying out the plan*, and *looking back*. Since students acquire new knowledge in problem solving situations, the teacher has a very important role in emphasizing problem solving in class.

New Math Movement

To understand the emphasis on problem solving in the SMSG textbooks, it is important to have a background of the context in which the curriculum materials were developed. In the 1950s, there was a public outcry in the United States about declining standards and a general dissatisfaction with the teaching of mathematics in schools (see Kilpatrick, 1992). The philosophy for the reform was also strongly advocated in the first SMSG Newsletter of March 1959: “We need an improved curriculum which will offer students not only the basic mathematical skills but also a deeper understanding of the basic concepts and structure of mathematics (cited in Kilpatrick, 1970, p. 125).”

The SMSG was initiated in the spring of 1958 as a nationwide project for the improvement of teaching of mathematics in schools (Kilpatrick, 1970). The dominant theme was based on the view that curricula in schools should emphasize the axiomatic and structural aspects of mathematics rather than its applications and it was believed that an emphasis on structure would aid memory, understanding, and aid in transfer of learning. Thus, an attempt was made to bridge the gap between college mathematics and school mathematics. The influence from the discipline of mathematics came through the works of university and industrial mathematicians who played a major role in the new curriculum development projects.

The psychological rationale for teaching mathematics came from Bruner and Piaget. Bruner's structuralist approach was based on the investigations conducted by genetic epistemology theorists into the processes of concept formation (Howson, Keitel, & Kilpatrick, 1981). Bruner's theory was to orientate curricular reform to the structures of the scientific discipline and his concept of *spiral curriculum* was to aid in that direction. The essence of his theory was to transmit the scientific structures to pupils endowed with lower cognitive structures through the process of *discovery learning*. The spiral curriculum was to ensure that progression took place from lower to higher levels. The model for "New Math" teaching was *Socratic dialogue* guiding students to discovery of key concepts (NCTM, 1975). Besides Piaget, the primary mathematics curriculum was influenced by Dienes who worked extensively with a range of concrete aids to mathematical learning (Pitman, 1989). Bruner (1964) himself stated that knowing is a process not a product and had the student begin with problem solving for grasping the main ideas in the sequence of the curriculum. However, Howson, Keitel, and Kilpatrick claimed that the New Math approach was exclusively product oriented. Thus, from a philosophical point of view, mathematics was still in an absolutist paradigm, but there were moves towards a more fallibilist perspective of mathematics (Pateman, 1989).

Problem Solving Emphasis in the SMSG Textbooks for the Middle Grades

The SMSG textbooks for grades 5 and 6 have the same foreword and preface, unfortunately not addressed to the student. The expectations are set very high in the foreword: "This healthy fusion of the old and the new should lead students to a better understanding of the basic concepts and structure of mathematics and provide a firmer foundation for understanding and use of mathematics in a scientific society." In the preface, the authors claimed that: "There is less emphasis on rote learning and more emphasis on the construction of models and symbolic representation of ideas and relationships from which pupils can draw important generalizations." Furthermore, the authors also added that the simple ideas are introduced early and then they are frequently examined to explore them more fully and to apply them in solving problems.

The foreword and the preface set the tone for what is contained in the textbooks. There is a great emphasis on the structure of mathematics and on the understanding of mathematical ideas. Most of the chapters start with a brief explanation (sometimes referred to as exploration) followed by a set of exercises and then there are some more exercises at the end of the chapter. The NCTM (1963) report on the SMSG described the problems for grades 4-6 as primarily demonstrating the application of a process or a concept. "Relatively few of the problems posed were the kind which made the learner aware of the need for a new mathematical process and the concomitant conceptual development." (NCTM, 1963, p. 34). The NCTM report further stated that the SMSG program gave more attention to mathematical structure than most elementary programs and then divided this structure into units that were often unrelated, or incidentally connected. Writing about the problems in the "modern" texts of the 1960s, Polya (1966) claimed that: "Modern textbooks often have chapters full of new terms and symbols which remain unrelated to the experience and background of the student and of which, therefore, the student cannot make serious use. And so the problems at the end of the chapter are particularly flat routine problems, most of them just vocabulary questions" (pp. 126-127). If the problems were unrelated to the experience of the children, how were they to make sense of the mathematics they had to study? This clearly shows that the problem solving process was not a focus, but students were simply expected to solve problems.

Evaluation of student performance also provided some evidence of their general ability, and certainly some insights into their corresponding problem solving ability. One of the results of the *National Longitudinal Study of Mathematical Abilities* (fall of 1962 to spring of 1967) carried out by the SMSG, showed that student performance was related to the use of various textbooks (Howson, Keitel, Kilpatrick, 1981). For, example at the end of grade 8, students who had a conventional textbook tended to perform better than any of the groups using modern textbook series. But students using modern textbooks had a better understanding of number properties. The authors added that students tended to learn what was emphasized in the textbooks they used and nothing else.

I noticed that generally, some of the problems in the students' textbooks for grades 5 and 6 are fairly well illustrated with diagrams, tables, and charts but they lean heavily on the conceptual side. I have reviewed the texts for grade 7 as well (part of junior high) and the problems are set in very much the same way. The problems are generally of the "closed type", having only a single solution. None of the problems are open-ended or requires a group attempt. In the solved examples, sometimes there are demonstrations of solution of problems by two methods, but this is not really emphasized in the problems that the students have to solve. There is a fair amount of computation that students have to do, given that a calculator was not available at that time. The students were given no directions about how to proceed or strategies that they may use. There seems to be a heavy reliance on the authority of the teacher for directions and thus it would be difficult for a student to do independent work all by herself or himself. Accordingly, I think the emphasis of the writers of the texts was not so much on the problem solving as a process, but more so as a skill whereby the product of the problem solving process was considered more valuable than the process itself. This may be partly due to the lack of research or the different focus of research around that time. Tom Romberg, in an interview by Kieran (1994), referring to the problem solving emphasis in the 1960s, claimed that past research on problem solving did not have a good theory of representation or of how one manipulated these representations. A point also made by Schoenfeld (1994) was that a focus on statistics (quantitative studies) made researchers avoid a deeper look at the process of problem solving. Thus, a lack of a good psychological theory underlying research on problem solving could have influenced the writers of the SMSG texts in the 1960s.

It should be noted that learning by discovery, which was advocated by the SMSG texts, was a controversial issue (see Ausubel, 1971). The controversy centered on the amount and the kind of guidance that ought to be provided to students in the learning situation, as those favoring learning by discovery advocated minimal teacher guidance and maximal opportunity for exploration and trial and error on the part of the student (Shulman, 1970).

Problem Solving and the Standards Movement

The early 1980s was a period of crisis, for it became clear by that time that the *Back-to-Basics* movement, following the New Math era, was a failure (Schoenfeld, 1992). Schoenfeld added that the curricula of the previous decade focused on rote mechanical skills and it produced a generation of students, who for lack of exposure and experience, performed dismally on measures of thinking and problem solving. The advent of computers and calculators also raised the question whether school children should spend less time on routine computational exercises, thereby releasing more classroom time for teachers to involve children actively on problem solving tasks (Pitman, 1989). The document *Agenda for Action* was published in 1980 by the NCTM. It emphasized problem solving as the main focus of school mathematics and thus, researchers around that time turned increasingly to more process-oriented studies.

One of the goals set by the NCTM (1989) for K-12 mathematics education was that students become mathematical problem solvers, if they were to become productive citizens. It was perceived as important that students solve problems not only alone, but also by working cooperatively in small or large groups. Problems had to be varied, with some being open-ended and some in more applied contexts. Regarding the grades 5-8 curriculum, the NCTM (1989) standards stated that it should take advantage of the expanding mathematical capabilities of middle school students to include more complex problem situations involving topics such as probability, statistics, geometry, and rational numbers. The standards also suggested that some of the problems had to be more demanding, requiring extended effort from the students. It was also claimed that students had to make full use of available technology as problem-solving tools and that they had to learn to work cooperatively on selected problems. Students were encouraged to communicate their solutions. In middle grades, the standards reiterated that problem solving should promote mathematical learning, and because of their expanding mathematical capabilities, problems should be chosen from different areas of mathematics. Students had to be encouraged to find multiple solutions to problems and make appropriate use of technology. The standards also mentioned that students had to reflect on their problem solving and consider how it might be modified, elaborated, streamlined, or clarified.

Problem Solving Emphasis in CMP Textbooks of the Middle Grades

The Connected Mathematics Project (CMP) was developed at Michigan State University and funded by the National Science Foundation (NSF). I have examined the texts for grades 6-8. The text is user-friendly and the writers of the texts address directly the students about the main ideas that would be developed in the texts in a section called *Mathematical Highlights*. Unlike the SMSG writers, the writers of the CMP texts do not elaborate on the mathematical concepts before introducing the problems. The introductory problems are set as activities within which the students are supposed to discover the mathematical principles and then there are follow-up activities for covering the content to a greater depth. Some of the activities can be done individually, but most of them require the students to work in small groups. Skills development is not neglected. There are problems set as Applications (A), Connections (C), and Extensions (E) and students are expected to use calculators. These ACE problems are ordered in terms of difficulty within each of the three parts. The problems are well illustrated with diagrams, graphs, and charts and set with various colors. The use of different forms of representations helps students to grow in their ability to reason effectively with information and in their ability to move flexibly among these representations. After each chapter there is a section on *Mathematical Reflections*, whereby the student is given the opportunity to reflect on the main ideas developed in the chapter. Unlike the SMSG textbooks, which are set in two volumes covering more than 300 pages each, the CMP textbooks at each level are set in small booklets having around 80 pages. The problems in the CMP texts are set in everyday life contexts in a variety of situations which are of relevance to the students. Many of the problems cut across other disciplines at the middle school level and are quite varied, catering for a wider range of abilities of the students. Thus, the teacher can choose from the ACE problems those that are appropriate at the level of performance of a particular student.

A Comparison of SMSG and CMP Textbooks for Emphasis on Problem Solving

There are very few similarities between the two sets of materials. Both have some emphasis on skill development and certainly on understanding, but otherwise there are more differences. Compared to the SMSG curriculum, the CMP textbooks have benefited from many years of research in the field of problem solving. Even though there are fewer studies in problem solving now at various grade levels, the major thrust that the standards movement brought about has been significant. Problem solving is looked into more as a process now than it was in the SMSG textbooks. It is not only research, but also the use of technology which has been very influential in giving a broader view to problem solving in the recent curriculum materials. Calculators and computers have reduced the reliance on computation and released that time for more focus of other aspects of problem solving in class, compared to the SMSG materials. The SMSG materials focused much more on individual solutions to problems, whereas the recent curriculum materials such as the CMP has a greater focus on cooperative learning in small groups. The problems that students have to solve now are grounded in applications of interest to them in their everyday life unlike those in the SMSG curriculum, where the focus was much more on the content to be studied. The problems that students have to solve now are more centered around activities or mathematical tasks compared to those of the 1960s. Students now benefit from research on metacognition, how to monitor one's own thinking in problem solving situations. As research on metacognition started around the mid-1970s, the SMSG curriculum did not benefit from this. The CMP materials have a section on *Mathematical Reflections* at the end of each chapter, which is not simply a summary. This helps the students to focus again some of the main ideas they developed in the chapter.

School textbooks generally have a short life-span and so they are products of a particular time period; the SMSG and the CMP textbooks are no exception. Problem solving is only one criterion for comparing the two sets of curriculum materials developed so many years apart. Perhaps the most significant aspect of a textbook is how it is used by the teacher in class. So, how a teacher emphasizes problem solving using the textbooks matters the most.

References

- Ausubel, D. P. (1971). Limitations of learning by discovery. In D. B. Aichele, & R. E. Reys (Eds.), *Readings in secondary school mathematics* (pp. 193-209). Boston, MA: Prindle, Weber & Schmidt, Inc.
- Bruner, J. S. (1964). *Toward a theory of instruction*. Cambridge, MA: Harvard University Press.
- Connected Mathematics Project. (1996). *Bits and pieces I, Grade 6* (Student edition). Palo Alto, CA: Dale Seymour Publications.
- Connected Mathematics Project. (1997). *Accentuate the negative, Grade 7* (Student edition). Palo Alto, CA: Dale Seymour Publications.
- Connected Mathematics Project. (1998). *Clever counting, Grade 8* (Student edition). Palo Alto, CA: Dale Seymour Publications.
- Howson, G., Keitel, C., & Kilpatrick, J. (1981). *Curriculum development in mathematics*. New York: Cambridge University Press.
- Kieran, C. (1994). Doing and seeing things differently: A 25-year retrospective of mathematics education research on learning. *Journal for Research in Mathematics Education*, 25(6), 583-607.
- Kilpatrick, J. (1970). Cognitive theory and the SMSG program. In K. Kramer (Ed.), *Problems in the teaching of elementary school mathematics* (pp. 124-130). Boston, MA: Allyn and Bacon, Inc.
- Kilpatrick, J. (1992). A history of research in mathematics education. In D. A. Grouws (Ed.), *Handbook of research on mathematics teaching and learning* (pp. 3-38). New York: Simon & Schuster Macmillan.
- Krulik, S., & Rudnik, J. A. (1980). *Problem solving: A handbook for teachers*. Boston: Allyn & Bacon.
- National Council of Teachers of Mathematics. (1963). *An analysis of new mathematics programs*. Washington, DC: Author.
- National Council of Teachers of Mathematics (1975). *Overview and analysis of school mathematics grades K-12*. Reston, VA: Author.
- National Council of Teachers of Mathematics. (1980). *An agenda for action: Recommendations for school mathematics in the 1980s*. Reston, VA: Author.
- National Council of Teachers of Mathematics. (1989). *Curriculum and evaluation standards for school mathematics*. Reston, VA: Author.
- Pateman, N. A. (1989). *Teaching Mathematics: A tantalising enterprise*. Geelong, Vic, Australia: Deakin University Press.
- Pitman, A. (1989). Mathematics education reform in its social, political and economic contexts. In N. F. Ellerton, & M. A. Clements (Eds.), *School mathematics: The challenge to change* (pp. 102-119). Geelong, Vic, Australia: Deakin University Press.
- Polya G. (1957). *How to solve it?* (2nd ed.). New York: Doubleday & Co.
- Poly, G. (1966). On teaching problem solving. In E. G. Begle (Ed.), *The role of axiomatics and problem solving in mathematics* (pp. 123-129). Boston, MA: Ginn and Company.
- Schoenfeld, A. H. (1992). Learning to think mathematically: Problem solving, metacognition, and sense making in mathematics. In D. A. Grouws (Ed.), *Handbook of research on mathematics teaching and learning* (pp. 334-370). New York: Macmillan Publishing Company.
- Schoenfeld, A. H. (1994). A discourse on methods. *Journal for Research in Mathematics Education*, 25(6), 697-710.
- School Mathematics Study Group. (1965). *Mathematics for the elementary school: Grade 5, Parts I & II* (Student's text). Stanford, CA: Stanford University.
- School Mathematics Study Group. (1965). *Mathematics for the elementary school: Grade 6, Parts I & II* (Student's text). Stanford, CA: Stanford University.
- School Mathematics Study Group. (1960). *Mathematics for the junior high school: Volumes 1 and 2, Parts I & II* (Student's text). New Haven, CT: Yale University Press.
- Shulman, L. S. (1970). Perspectives on the psychology of learning and the teaching of mathematics. In K. Kramer (Ed.), *Problems in the teaching of elementary school mathematics* (pp. 81-96). Boston, MA: Allyn and Bacon, Inc.