Comparing mathematics education traditions in four European countries: The case of the teaching of percentages in the primary school

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1. Introduction

This paper presents a small-scale videobased, comparative study of the teaching of percentages in four European countries. This study is situated within the METE-project (*M*athematics *E*ducation *T*raditions in *E*urope), which aims at a comparison of mathematics education in the upper primary (grades 5 and 6) and the lower secondary school (grades 7 and 8) in Flanders, England, Hungary and Spain. Major criteria for the selection of these countries were the diverse geographical location in Europe, their different socio-economical status, and their various performances on large-scale studies, such as TIMSS and PISA. In each country sequences of four or five lessons relating to several topics of the mathematics curriculum were videotaped in the same classroom: percentages and polygons in the upper primary school (age 10-12), and polygons and linear equations in the lower secondary school (age 12-14).

In this chapter, we will restrict ourselves to a discussion of the percentage part of this comparative study. The next section describes a perspective on the teaching of percentages that was derived from the literature on mathematics education in general and the teaching of percentage in particular, and that relates to its objectives, conceptual aspects, and didactic tools. In the third section, the aims and methodology of the METE-project – and more specifically the percentage part – is outlined. The main outcomes of the study are presented in the fourth section. We conclude by discussing some principal ideas and the constraints of our approach.

2. A perspective on the teaching of percentages

The available research relating to the teaching of percentages is rather restricted. Nevertheless we have made an attempt to frame the observed lesson series in the four countries within a perspective on the teaching of percentages that is derived from the recent literature, and that draws inspiration from a realistic view on mathematics education. In this section, this perspective will be outlined focusing on the objectives of percentage instruction, conceptual aspects of percentages, and some didactic tools used in teaching percentages.

2.1 Objectives

The instruction of percentages serves different goals, namely computational, conceptual, and applicational goals. First, attention should be paid to computational goals (Van den Heuvel-Panhuizen, 1994). In other words, students should master one (ore more) procedure(s) to compute percentages. The attainment of this objective results in the development of procedural knowledge. Although this is an important goal, it is not the only objective to focus on. Indeed, there is some criticism on the teaching of percentages that "is primarily focused on procedures and recall instead of getting a real understanding of percentage" (Van den Heuvel-Panhuizen, 1994, p. 350).

Therefore, a second important goal is students' development of a deep understanding of the concept "percentage" (Van den Heuvel-Panhuizen, 1994). They need to acquire insight in the key features of percentages so that meaningful learning is realised. To meet this objective the acquisition of a good and consistent conceptual knowledge system is required.

Finally, a third goal is the development of skills to apply percentages in all kind of (meaningful) situations. Students should acquire adaptive expertise allowing them to apply procedures flexibly to new, as well as familiar tasks, and to solve percentage problems in a variety of ways. Important for the development of adaptive expertise is the acquisition of a well-structured knowledge base wherein the concept of percentages is related to prior knowledge and to other mathematical entities. This also requires that students' procedural (objective 1) and conceptual (objective 2) knowledge get interconnected (Baroody, 2003).

In the next section, we will describe some characteristics of percentages that should be addressed in the classroom to enable students to acquire a deep understanding of percentages.

2.2 Conceptual aspects of percentages

Through solving appropriate tasks and problems students should discover that a percentage expresses a relation between two numbers or quantities by means of a ratio. They should become aware that percentages are always related to something and have, therefore, no meaning without taking into account to what they refer (Van den Heuvel-Panhuizen, 1994). For example: “You need 50% correct answers to succeed. Loes solved 14 tasks wrongly. What do you think? Can we congratulate Loes or not?”. Since the number to which the percentage is referring is missing, no judgement can be made about Loes’ performance.

A second characteristic of percentages is that they describe a fixed situation representing how different kinds of substances are related to each other. This means that the size of the whole has no influence on the percentages of the substances. The following problem illustrates this characteristic: “Black currant jam, which contains 60% of fruit, is sold in large (450g) and small pots (225g). Someone forgot to put the percentage of fruit on the small pot. Fill in this missing information. Explain your strategy for finding this percentage. How many grams of fruit does each pot contain? The large one contains …? The small one contains …? Show how you got your answers.” (Van den Heuvel-Panhuizen, 1994, p. 357). By means of such examples, students can overcome the (potential) misconception that percentages change linearly with the size of the whole.

A third characteristic is that the reduction or adding up of percentages has a non-linear character. For example: a whole plus 20%, plus 30%, is not the same as the same whole plus 50%. Parallel with that, the decrease or increase of a part behaves asymmetric if it is expressed by percentages. The reason for the asymmetry in adding and removing parts is the fact that the reference amount changes (in the latter case the reference amount is larger). But, the part that was added or removed stayed the same.

Finally, percentages can be used to describe two different types of situations. First, they can describe the substances of a whole. In that case, they describe the size of a part in relation to the whole (Streefland, 1995). For example, to bake bread you need 73% flour, 25% water, 1% yeast, and 1% butter. Second, percentages can describe situations about a whole that is increased or decreased with a part (Streefland, 1995). For example, “100 visiting cards including V.A.T. cost €10. What’s the price of the visiting cards excluding the V.A.T.?”

2.3 Didactic tools

To initiate the teaching of percentages, the teacher can use a lot of everyday situations that are intelligible for students. In fact: “Teaching percentages does not start when the name percentage is mentioned for the first time, but it has its roots in all kinds of ‘so-many-out-of-so-many’ situations that have been dealt with long before, at least in everyday situations” (Van den Heuvel-Panhuizen, 1994, p. 353-354). Indeed, students already have a lot of informal knowledge, for example concerning “anchor” percentages, such as 25, 50 and 100 percent and their equivalent fractions (Streefland, 1998). Therefore, attention should be paid to the relationship of the formal concept “percentage” to students’ informal prior knowledge.

To induce meaningful learning in children, especially to foster adaptive expertise (see section 2.1) strong emphasis should be given to the relationship of the concept percentage to other mathematical entities. Some connections are self-evident, such as with fractions, ratios, decimal numbers, … (Streefland, 1995).

In general, the instruction of percentages can be conceived as progressive mathematisation in which models and concrete materials have an important role to play. Van den Heuvel-Panhuizen (2003) defines models as “representations of problem situations, which necessarily reflect essential aspects of mathematical concepts and structures that are relevant for the problem situation, but that can have different manifestations” (p. 13). With respect to percentages, some models are very common to use during the instruction, such as the pizza model or pie chart, the ratio table, and the bar model. Not so common, although very useful models for the instruction of percentages are the elastic percent meter and the slide-slip. The simultaneous use of different models should be encouraged, since it can foster students’ conceptual development. Powerful models have at least two important characteristics. First, they are rooted in realistic and imaginable contexts. Second, they are sufficiently flexible to be applied on a more advanced and general level. If models meet those requirements, they can bridge the gap between the informal understanding connected to the “real” and imagined reality, on the one hand, and the understanding of formal systems, on the other hand. While shifting from the informal to a more formal level, models undergo a change from “model of” into “model for”: at the beginning of instruction, models are very closely connected to the problem situation, but they gradually evolve to models that are generalised over contexts and can be applied to new and unfamiliar situations. Applied to the bar model, during the instruction of percentages it can shift in function and in form. As a result of the change in function the bar model evolves from a concrete context-connected representation into a more abstract representational model that is applicable to divergent contexts and that also serves as a calculation model. The change in form results into the double number line, which is simpler, easier to use and more flexible (Van den Heuvel-Panhuizen, 2003).

3. Aims and methodology of the comparative study

3.1 Aims

A socioconstructivist view on learning and teaching constitutes the background of the study: learning is conceived as a social construction mediated by teaching, and can only be understood if one takes into account the sociocultural setting in which learning takes place (McCaslin & Hickey, 2001). Educational studies from a socioconstructivist perspective focus on the analysis of learning-in-context. They try to understand the world of signification and meaning in which students act and learn guided by instruction (Cobb & Bowers, 1999).

Similarly, the present study tries to understand some mathematical practices within their specific educational context. It is designed to identify characteristic patterns of classroom activity in general and effective approaches to the teaching of percentages, in the age range of 10-12, in particular. Moreover, the study aims at framing the distinctive features of percentage instruction in the participating countries within current thinking relating to teaching and learning percentages (see section 2). It does, however, not aim at evaluation and generalisation of the different educational practices.

3.2 Methodology

As mentioned before, the focus of the project was on comparing sequences of four or five lessons relating to the teaching of percentages, polygons, and linear equations. However, in view of analysing the videotaped lessons the international research team was confronted with the challenge to develop and try out the necessary instruments. Therefore, during the first phase of the project, which lasted more than one year, live observations by members of the four country teams took place in all countries. Spread over one week a range of lessons was observed, and at the same time videotaped. The teams collaboratively worked on the development of instruments for describing mathematics teaching-learning practices through discussions starting from watching the videoregistration of the lessons observed. This iterative process resulted in two instruments: a lesson synthesis sheet and a lesson coding scheme (see the general paper in this symposium (Andrews, 2005) for a detailed description). The percentage lessons were analysed according to these two instruments. The analyses were done by the team of the country in which the lessons were videotaped. A sample of all project lessons was scored by two independent coders, and an interscorer-reliability of .80 was obtained.

4. Outcomes

This section reviews both the differences and the similarities in the distinct approaches of the four countries to the teaching of percentages. First, we will outline a comparative analysis on the distinct categories of both the lesson synthesis sheet (pedagogic activities and social activities) and the lesson coding scheme (mathematical focus, mathematical context, didactics, and materials). Thereafter, we will use the results of that comparative analysis to frame the sequence of lessons in the current thinking on teaching percentages as described in section 2 (the objectives, the conceptual aspects, and the didactic tools). We will use the country name to label the sequences of lessons; however, it is obvious that the restricted dataset does not allow to generalise the results of that specific approach to the teaching of percentages in general in each country.

4.1 Analyses of the lesson synthesis sheets

4.1.1 Pedagogic activities

All videotaped percentage lessons were analysed in terms of the seven distinguished pedagogic activities: theory or conceptual development, working on problems or tasks, reporting solutions to problems or tasks, introducing a problem or activity, homework-related activities, task-related management, and non task-related management. The distribution of these pedagogic activities in the four participating countries is represented in Figure 1.

First, in all countries a substantial amount of time was spent on working on problems or tasks. Especially in Flanders and England the teacher involved her students a lot in solving routine and non-routine tasks. Second, reporting of procedures to solve those tasks and solutions took a substantial amount of time in the Hungarian and English lessons. It occurred less in the other sequences of lessons. Third, there is one great difference in the time that was spent on the introduction of new

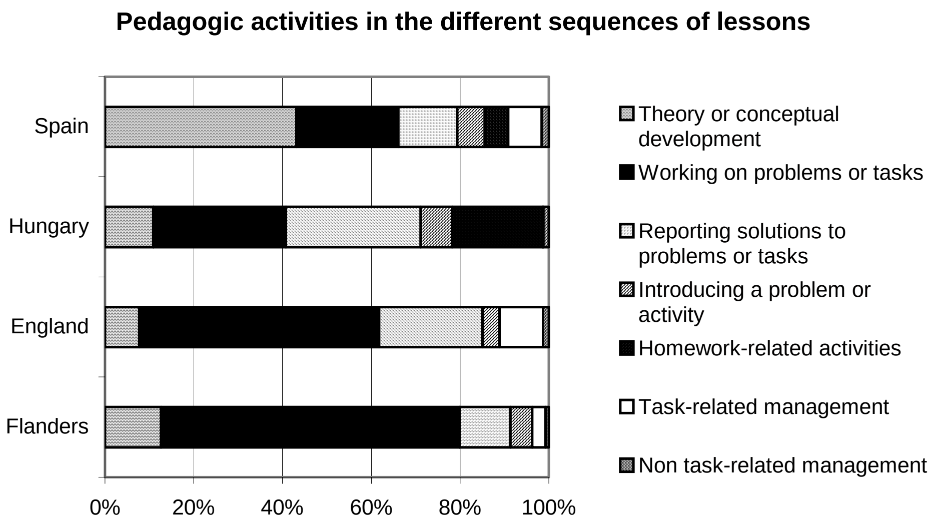

Figure 1: Comparative analysis of the pedagogic activities

concepts, or the activation of prior knowledge. The Spanish teacher spent a lot of time on that activity, whereas it occurred less in the other lessons. Fourth, homework-related activities, including the distribution of homework, giving instructions about it, or discussing about the solutions of the homework tasks, was totally absent in the Flemish and English approach to the teaching of percentages. Also the Spanish teacher did not spent a lot of time to that activity. Contrary to those low scores, more than one fifth of the time of the Hungarian lessons was spent on homework-related activities. Fifth, in all country approaches about 5% of the total lesson time was spent on the introduction of a problem or activity. Sixth, overall not much lesson time was used up for practical instructions related to the solving of the tasks, such as the distribution of worksheets or equipment, the formation of groups, etc. Finally, the occurrence of non task-related activities, i.e. practical interventions that have nothing to do with mathematics, was in almost every sequence of lessons absent.

4.1.2 Social activities

The videotaped percentage lessons were also analysed with regard to the time that was spent on the different social activities: whole class activity, individual activity, paired activity, and group activity. Figure 2 represents the frequency of these different social activities in the different sequences of lessons.

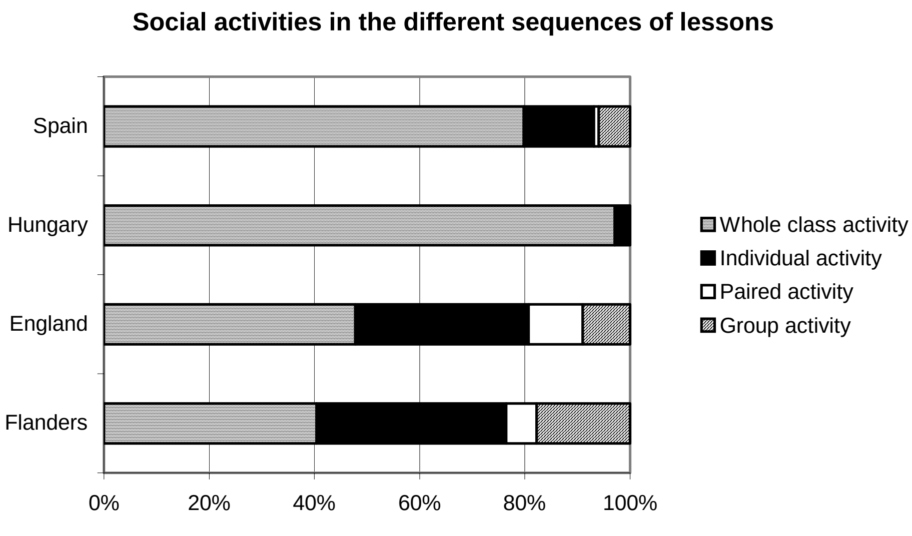

Figure 2: Comparative analysis of the social activities

Whole-class activity is clearly the dominant form of organization in all four lesson sequences. But as Figure 2 shows there are substantial differences between the different approaches. In the Hungarian lessons almost all the time was used up by this category. Also in the Spanish lessons, the whole class worked on the same activity at the same time during a very large part of the lessons. This was less the case in the two other sequences of percentage lessons. Working individually on problems or tasks had the second highest frequency in all countries, but also here large differences were observed. Third, working in groups occurred most frequently in the Flemish lessons. It was observed much less in the English and Spanish lessons, whereas it was totally absent in the Hungarian lessons. The same holds true for activities in pairs that did not take place in the Hungarian lessons. Also in the Spanish lessons it was almost absent; it occurred somewhat more during the Flemish lessons, and it was most present in the English lessons.

4.2 <u>Analyses of the lesson coding schemes</u>

4.2.1 Mathematical focus

The sequences of percentage lessons in the four participating countries were analysed in terms of the seven different mathematical foci: conceptual, derivational, structural, procedural, efficiency, problem solving, and reasoning. The frequencies of the foci in each lesson sequence were compared and are presented in Figure 3.

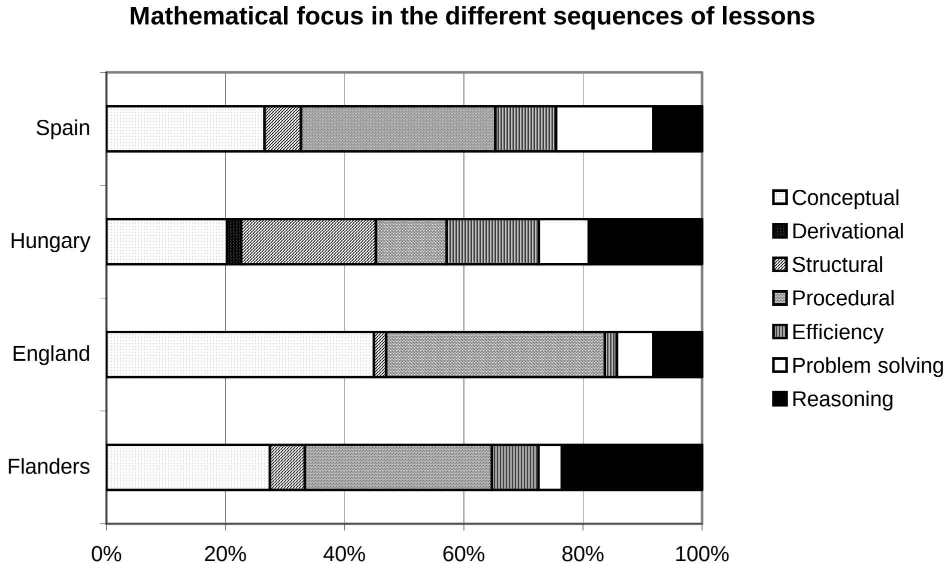

Figure 3: Comparative analysis of the mathematical focus

In all four countries the two strongest foci were the procedural and the conceptual ones. A wide variation was observed in the codings for the other mathematical foci.

The data were subjected to a Mann-Whitney U-test in order to determine the effect of the specific country approaches on the mathematical focus. This test contrasts the mean score on each specific focus in one country approach with the average score on that focus in the other three country approaches. The results are shown in Table 1. (We accepted a maximum level of $p = .05$ for statistical significance. The direction in which the focus of one approach differs statistically from the other approaches can be read off from Figure 3.)

These results reveal that the sequence of the Hungarian lessons significantly differed for the major part of the mathematical foci from the other lesson sequences. Compared to other country approaches, the Hungarian lessons had a stronger derivational, structural, efficiency, and reasoning focus, while there is less emphasis on procedural knowledge and skills. Statistically significant differences for the English lessons were found for the conceptual (i.e., a stronger conceptual focus than in the other country approaches) and the efficiency focus (i.e., a weaker efficiency focus than in the other country approaches). The approaches of the Flemish and the Spanish teacher show no significant differences with the other approaches.

4.2.2 Mathematical context

The tasks involved in the four country approaches to teach percentages were analysed with regard to their relatedness to the real world and the genuineness of their data. Figure 4 presents the frequency of

Table 1: Results of the Mann-Whitney U-test for mathematical focus

Mathematical focus		Flanders	England	Hungary	Spain
Conceptual	Z	-1.612	**-1.961**	-0.260	-0.504
	p	0.107	**0.050**	0.795	0.645
Derivational	Z	-0.904	-0.779	**-2.351**	-0.779
	p	0.366	0.436	**0.019**	0.721
Structural	Z	-1.427	-1.423	**-3.329**	-0.626
	p	0.154	0.155	**0.001**	0.574
Procedural	Z	-0.101	-1.414	**-2.221**	-0.870
	p	0.920	0.157	**0.026**	0.442
Efficiency	Z	-0.878	**-2.226**	**-2.996**	-0.056
	p	0.380	**0.026**	**0.003**	0.959
Problem	Z	-1.980	-0.505	-0.625	-1.965
Solving	p	0.048	0.613	0.532	0.061
Reasoning	Z	-0.608	-1.637	**-2.431**	-1.637
	p	0.543	0.102	**0.015**	0.127

the four subcategories: real world fabricated data, not real world fabricated data, real world genuine data, and not real world genuine data.

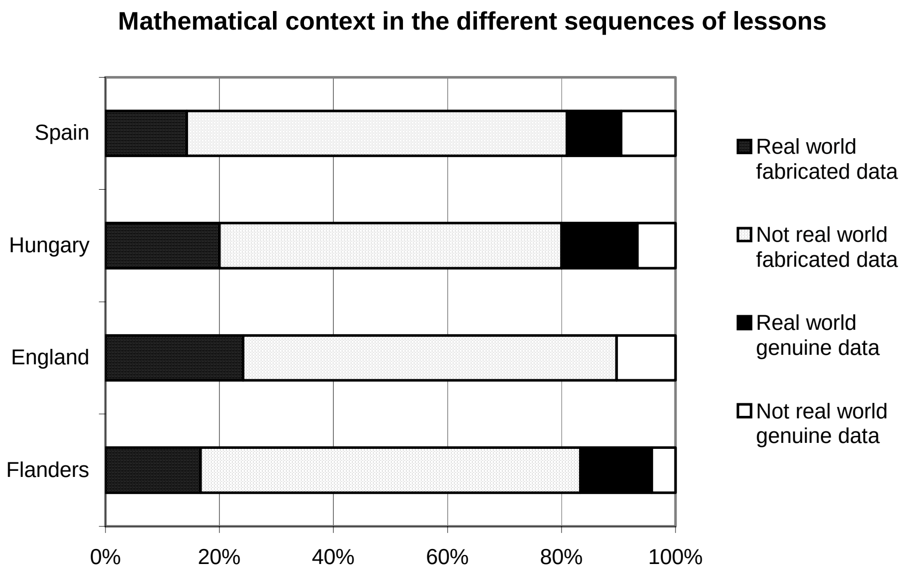

Figure 4: Comparative analysis of the mathematical context

Figure 4 shows a very homogeneous picture for the use of different contexts over the four approaches. As appears from Figure 4, most of all activities (approximately two thirds) were embedded in a context that was not explicitly related to the real world and that consisted of fabricated data. The second most used context was related to the real world and based on fabricated data. Real world genuine data were absent in the English approach, while the other teachers used them in almost the same frequency. Not real world genuine data were not frequently present in all approaches. The results of the Mann-Whitney U-test on these data are presented in Table 2.

As shown in Table 2, there are no statistically significant differences in the use of mathematical contexts between the analysed approaches to teach percentages.

4.2.3 Didactics

Next, we analysed the didactic strategies of the teachers from the four countries. The following didactic strategies were distinguished: activating prior knowledge, exercising prior knowledge, explaining, sharing, exploring, coaching, assessing/evaluating, motivating, questioning, differentiation. Their frequency across the different approaches is compared in Figure 5.

Table 2: Results of the Mann-Whitney U-test for mathematical context[1]

Mathematical Context		Flanders	England	Hungary	Spain
RWFD	Z	-0.156	-1.007	-0.260	-1.119
	p	0.876	0.314	0.795	0.327
NRWFD	Z	-0.255	-1.209	-0.765	-0.110
	p	0.799	0.227	0.444	0.959
RWGD	Z	-0.118	-1.521	-1.647	-0.380
	p	0.906	0.128	0.099	0.798
NRWGD	Z	-0.339	-0.365	-0.135	-0.146
	p	0.735	0.715	0.892	0.959

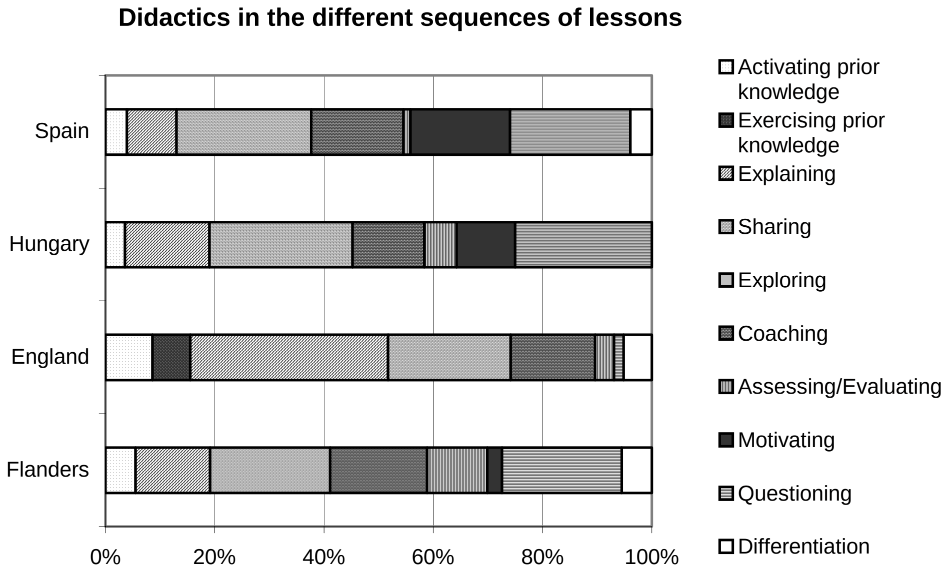

Figure 5: Comparative analysis of the didactics

Figure 5 reveals that four didactic strategies were used quite frequently in all country approaches: sharing, questioning, explaining, and coaching. The others were used (much) less frequently. A wide diversity was observed in the codings of these didactics over the different country approaches. Table 3 shows the results of the Mann-Whitney U-test.

Table 3 shows that the English approach was responsible for most of the observed differences in didactics between countries. More specifically, the English teacher activated prior knowledge more, exercised prior knowledge more, explained more, motivated less, and questioned less than in the other country approaches. The two other observed deviations were the Hungarian teacher's lesser attention to differentiation and the Spanish teacher's greater attention to motivating pupils.

4.2.4 Materials

The materials used by the teachers and the students to support the teaching-learning process were analysed in the four approaches. Table 4 shows the results of this (comparative) analysis of the fourteen didactic tools.

In general, the data presented in Table 4 reveal that teachers frequently use the board during their instructional practices. To the contrary, real world materials were only used to a small extent by both the teacher and the students. Some materials were never used by the teacher, such as worksheets, practical equipment, calculators, real world materials, and answer books. Students also did not use computers and practical equipment.

Besides, the results also show a wide diversity in the frequency of the materials that were used to support the teaching-learning process over the four country approaches. Flemish students frequently completed worksheets, while their Hungarian peers did not use them at all. Manipulatives were often

[1] RWFD = real world fabricated data, NRWFD = not real world fabricated data, RWGD = real world genuine data, NRWGD = not real world genuine data

Table 3: Results of the Mann-Whitney U-test for didactics

Didactics		Flanders	England	Hungary	Spain
Activating prior	Z	-0.275	**-2.012**	-1.099	-0.533
knowledge	p	0.784	**0.044**	0.272	0.645
Exercising prior	Z	-1.144	**-3.450**	-1.144	-0.986
knowledge	p	0.253	**0.001**	0.253	0.574
Explaining	Z	-1.480	**-2.529**	-0.408	-1.374
	p	0.139	**0.011**	0.683	0.192
Sharing	Z	-1.560	-0.217	-0.654	-1.192
	p	0.119	0.828	0.513	0.277
Exploring	Z	-1.612	-0.535	-0.620	-0.535
	p	0.107	0.593	0.535	0.878
Coaching	Z	-0.406	-0.383	-0.609	-1.475
	p	0.685	0.702	0.543	0.158
Assessing /	Z	-1.550	-0.954	-1.107	-1.909
Evaluating	p	0.121	0.340	0.268	0.101
Motivating	Z	-1.828	**-2.251**	-0.992	**-3.151**
	p	0.068	**0.024**	0.321	**0.001**
Questioning	Z	-0.203	**-2.951**	-1.522	-1.530
	p	0.839	**0.003**	0.128	0.158
Differentiation	Z	-0.886	-0.477	**-2.214**	-0.954
	p	0.376	0.633	**0.027**	0.442

Table 4: Comparative analysis of the materials used by the teacher and the students

Materials	Teacher				Students			
	F[2]	**E**	**H**	**S**	**F**	**E**	**H**	**S**
Text book	0%[3]	3%	9%	0%	9%	6%	18%	0%
Worksheet	0%	0%	0%	0%	35%	9%	0%	20%
Game	0%	3%	23%	0%	0%	3%	23%	5%
Manipulatives	4%	0%	0%	5%	43%	9%	9%	25%
Practical equipment	0%	0%	0%	0%	0%	0%	0%	0%
Overhead projector	0%	13%	18%	0%	0%	3%	14%	0%
Computer	0%	3%	0%	0%	0%	0%	0%	0%
Calculator	0%	0%	0%	0%	17%	9%	0%	0%
Real world materials	9%	3%	0%	0%	13%	0%	0%	5%
Pupil whiteboards	0%	0%	0%	0%	0%	53%	0%	0%
Board	43%	59%	73%	75%	9%	3%	82%	5%
Coloured writing materials	0%	22%	23%	0%	4%	3%	0%	0%
Answer book	0%	0%	0%	0%	9%	0%	0%	0%
Display material	13%	6%	14%	10%	0%	0%	14%	10%

used by the Flemish and Spanish, whereas this was clearly less the case in the English and Hungarian lessons. Both the English and Hungarian teacher and students frequently used the overhead; this never occurred in the Flemish and Spanish lessons. Only the Flemish and English students used their calculator. As already mentioned above, real world materials were not frequently used in the different approaches; only in the Flemish lessons a substantial amount of real world materials was used. The English students frequently wrote down their solution(method)s on their whiteboards, whereas the Hungarian students worked frequently on the blackboard.

4.3 Framing the four approaches within the current perspective on teaching percentages

The quantitative results obtained by comparing the different approaches to the teaching of percentages with respect to their mathematical focus, mathematical context, didactics, and materials, enable us to

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[2] F = Flemish approach, E = English approach, H = Hungarian approach, S = Spanish approach

[3] The percentages given in this table were obtained by dividing the number of episodes in which the material was used by the total number of episodes of each lesson. For each material, the obtained percentages were added up for all the lessons of that specific country approach.

frame these different approaches within the current perspective on teaching percentages, as described in section 2. Therefore, the four sequences of lessons will be qualitatively described according to their objectives, the conceptual aspects of percentages that are addressed, and their didactic tools. This qualitative description will be linked to the categories and subcategories of the lesson coding scheme.

5.2.1 Objectives

According to the perspective on teaching percentages presented in section 2, three major kinds of goals should be aimed at when teaching this topic: computational goals, conceptual goals, and applicational goals.

As shown by the analysis of the mathematical focus (see section 4.2.1), all four country approaches served computational goals. Indeed, the procedural focus was clearly present in all four lesson sequences, although significantly less in the Hungarian lessons. But we also observed substantial differences among the kind of procedures that were taught in the four countries. In the Flemish percentage lessons, students' computational process was initially supported by manipulatives (MAB-material). Gradually children learned to calculate percentages without these manipulatives. They applied the following procedure: dividing the given amount by hundred to calculate 1% of that amount; and multiplying that result by the percentage. Another, and more flexible, procedure Flemish students used to solve percentage tasks, was the arrow scheme (see Figure 6).

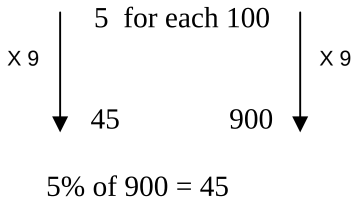

Figure 6: Example of the use of the arrow scheme

The English students learned two different procedures to compute successfully percentage tasks. A first procedure was to convert easy percentages (such as 10%, 20%, 25%, and 50%) into fractions simplified to their lowest term (e.g., with numerator one). This translation revealed by which number you should divide to calculate 10%, 20%, 25%, and 50% (which is respectively 10, 5, 4, and 2, since the equivalent fractions are $\frac{1}{10}$, $\frac{1}{5}$, $\frac{1}{4}$, and $\frac{1}{2}$). The "percentage-web" was another procedure that was applied to solve percentage tasks. In this procedure all percentages were related to 10%. For instance, 20% is the double of 10%. Consequently, 20% was calculated by dividing the whole by ten, and multiplying it by two. The more gifted children learned, in a similar way, to calculate more difficult percentages, such as 17.5% (e.g., 17.5% = 20% - 2.5% = (10% x 2) – (10% : 4)). Contrary to the Flemish students, the English students did not acquire the procedure to solve percentage tasks by dividing the whole by hundred to calculate 1% of that amount. Like the Flemish and English students, the Hungarian students also learned different procedures to solve all kind of percentage tasks. For instance, they translated percentages into their equivalent fractions (like in the English lessons), and they divided the whole by 100 (like in the Flemish lessons). Some other tasks were solved by means of estimation. For example, during the second lesson the teacher asked the students: "Could you show me – without any calculation – that 101% of the 90% of a number means a decrease of that number?". Students solved that exercise at the blackboard with a bar model. The Spanish teacher put strong emphasis on computational goals too. The students acquired a procedure to solve simple percentage tasks. Like in the English and Hungarian lessons, the Spanish students converted percentages into equivalent fractions in order to calculate the percentage. At the end of the lesson sequence, more difficult percentages were computed (e.g., 83% of 100). Gradually, students learned the skill to calculate percentages of an amount by multiplying the given percentage by 1% of the given amount.

The comparative analysis of the mathematical focus revealed that students' conceptual development was also a major concern of the instructional approaches to percentages in the four countries. The next section gives a more detailed description of the conceptual aspects that were dealt with in the lesson sequences.

Mainly in the Hungarian, and to a less extent in the Flemish approach, the students were frequently and strongly encouraged to apply known procedures flexibly and meaningfully to new and unfamiliar tasks. For instance, Hungarian students were faced with a wide variety of tasks each of which needed an appropriate solution procedure (e.g., the percentage and the solution were given, and the students had to find the original amount; or different parts of two circles were coloured, and the students had to calculate the percentage in degrees; or the students had to solve exercises that contained a combination of increase and decrease of a certain percentage). Some tasks were even solved in many different ways. This helped students to acquire adaptive expertise. For instance, students had to estimate percentages about the human population (e.g., the percentage illiterates); they were asked if a number increases or decreases after a combination of an increase and a decrease of a certain percentage. In the English and Spanish approach, adaptive expertise was not aimed at, since students were always asked to apply the same procedures to familiar tasks.

5.2.2 Conceptual aspects of percentages

Although, students' conceptual development of percentages was a major goal in all four sequences of lessons, the different aspects of percentages that, according to the current literature lead to a deep understanding of this mathematical concept, were hardly addressed in the four approaches. All teachers mainly focused on the "basic ideas" of the concept percentage, such as: a percentage always expresses something out of hundred, 100% equals the whole, etc. Moreover, the different formats of the tasks in the distinct approaches, reflected that percentages describe two kinds of situations: a part plus/minus a whole, and a part of a whole.

The most important and inherent characteristics of the percentage concept were revealed, most clearly, most deeply, and most systematically in the Hungarian approach. For instance, Hungarian students learned that percentages are always related to "something" and that they have no meaning without taking into account to what they refer. Moreover, they acquired the idea that percentages describe a fixed situation while representing how different percentages are related to each other (e.g., students had to represent the total human population by means of hundred pieces of paper; the teacher told them that the ratios in those 100 pieces and in the human population stay the same). Other exercises revealed the asymmetric nature of the increase and decrease of percentages. For instance:

T: 120% of its 80%. Is that an increase or decrease?
S: Increase … it remains the same.
T: Raise your hand if you think it increased. 120% of its 80%. Prove it! If I take 80% then it decreases. If I take its 120% then it increases. Who can prove it? Come on, Kinga.
S: [at the blackboard, with a bar model] We had 80%. We took away 20%. That increases by the 20% of this. But this is smaller than this, so its 20% is smaller too.So it does not increase back.
T: Good. Can you tell exactly, Kinga, what part is it that we got? If I think of a number. I think of 100.
S: Then it is 96.

Although it did not happen as explicitly and as systematically as in the Hungarian class, the Flemish teacher addressed some of these important conceptual aspects of percentages. For instance, class discussions revealed that percentages describe a fixed situation representing how different kinds of substances are related:

S: A pot of yoghurt with 9% fruit.
T: 9% fruit. What does that mean?
S: That there are 9% of 100 fruit in that.
T: 9% of the 100. I do not understand this very well. Who can explain it? Does that mean that there are nine pieces of fruit in that?
S: No.
T: Maybe 9 grams?
S: No.
T: Would it make a difference if it is a large or a small pot?
S: It depends … Yes, I think it would stay the same.

Moreover, pupils were also given a task to illustrate and explain the fact that percentages are always related to "something" and that they therefore have no meaning without taking into account the referent.

5.2.3 Didactic tools

As explained in section 2.3, teachers can use everyday situations that are intelligible for students when teaching percentages. However, the analysis of the mathematical context revealed that in neither approach a lot of real world genuine data was used (see Figure 4). Similarly, the analysis of the materials showed that real world materials were used only to a small extent in the Flemish, English, and Spanish lessons (see Table 4). Nevertheless, all four teachers, while introducing the concept of percentage, made explicit attempts to connect the newly introduced concept to students' informal everyday knowledge and experience. For instance, the situation of sales was used in every country to let the students explain what they already new about percentages.

To induce meaningful learning, especially to foster adaptive expertise, strong emphasis has to be put on the relationship of the concept percentage to other mathematical entities, like fractions, decimals… In all approaches, percentages were explicitly and strongly related to fractions. Interestingly, only the Hungarian teacher connected percentages with decimals and with the degrees of a circle (e.g., how many degrees equal one percent of a circle).

Furthermore, models and concrete materials have an important role to play in the instruction of percentages. Manipulatives, such as MAB-material and place-value-cards, were used in all four lesson sequences (see Table 4). Popular models to teach percentages were the ten by ten grid (in the Flemish, Hungarian, and Spanish approach), the pie chart model (in the English and Hungarian approach), and the arrow scheme (in the Flemish and Hungarian approach). Dynamic and powerful models such as the slide-slip and the elastic percent meter (see section 2.3), were not observed in any country approach.

5. Discussion

In this paper we made an attempt to describe differences in approaches to the teaching of percentages across four European countries: Flanders, England, Hungary, and Spain. This description is based on videotaped sequences of four or five consecutive lessons about (exactly) the same topic in one class that can be considered as typical (but evidently not perfectly representative) for the instructional approach in each of these four countries. Like for the research project as a whole, the goal was not to make a comparative evaluation of the quality of the instruction of percentages in these countries. Rather, we used the videotaped lessons to make an inventory of the variety in the different possibilities and traditions of the teaching of percentages within Europe, and to provide a critical reflection on these possibilities and traditions.

From a methodological perspective, we want to underline the need for using multiple methods to get a good understanding of the educational practice. As McCaslin and Hickey (2001) state: "Multiple methods no longer mean three quantitative instruments and their statistical relation; rather, multiple methods now routinely encompass qualitative work with quantitative" (p. 134). In that perspective, we complemented quantitative data obtained by means of a coding scheme consisting of four categories (mathematical focus, mathematical context, didactics, and materials) with qualitative descriptions of the analysed lessons involving their objectives, conceptual aspects, and didactic tools. This complementary use of quantitative and qualitative data has yielded an added value, in the sense that it enabled us to interpret the (quantitative) codings in light of the broader (qualitative) outlines of the lessons. For instance, the qualitative analyses of the procedures students acquired to solve percentage tasks facilitated a deeper understanding of the strong procedural emphasis in the four country approaches. Indeed, although all approaches focused (relatively) strongly on procedural skills, the qualitative analyses enabled us to grasp the similarities and differences in students' procedures in the distinct country approaches. However, in some cases, the qualitative analyses of the lessons also helped us to refine the outcomes of the quantitative outcomes on the lesson coding scheme. For instance, as the comparative analysis of the mathematical focus revealed (see Figure 3), the Spanish students seemed to be more involved with solving non-routine problems than in the other country approaches. Although this difference was not statistically significant (see Table 1), the qualitative analyses of the country lessons' objectives suggested that in the Spanish lessons the students did not acquire adaptive expertise, since they were always asked to apply the same procedures (routinely) to familiar tasks. Probably this gap between the outcomes of the quantitative and the qualitative analyses can be explained by the fact that the tasks in the Spanish lessons were experienced by the students and by their teacher as problem solving because of their rather low abilities, since the Spanish school was situated in a neighbourhood which is characterised by its low average socio-economic level, where the sale and consumption of drugs is commonplace. But, when compared to other country approaches (like Hungary and Flanders), the tasks of the Spanish lessons were rather easy and would by most mathematics educators not be labelled as illustrations of problem solving. Another valuable outcome of this project is the development of instruments for lesson analysis containing an internationally shared vocabulary, and possessing an acceptable interscorer-reliability. For sure the elaboration of these instruments was a time-consuming and difficult challenge.

Finally, it is desirable to supplement the outcomes of our analyses with other kinds of data gathering and data analysing techniques, such as interviews, questionnaires, and so on. In this study we necessarily restricted our analyses to observable teacher behaviour. We did not make any inferences about the learning processes and outcomes of the students involved in our investigation, because the methodology did not enable us to make such inferences.* Fien Depaepe is research assistant of the National Fund for Scientific Research – Flanders.

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