PISA Results and School Mathematics in Finland: strengths, weaknesses and future

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Abstract
The success of Finland in PISA is related to special strengths, among others the care of students with learning difficulties and the changes in school curriculum to meet with the international trends. Beside these strengths there are different types of weaknesses, which do not allow from achieving similar success in IMO, and do not offer universities and polytechnics with the wanted students. New development has started in 1995 to build a more balanced curriculum, and in 2004 the National Boards of Education published a new 'Curriculum Basics' to achieve this goal.

1. Background

Through history, old and new, Finnish people have faced different challenges to survival and success. In responding to these challenges, from time to time we got to hear about a Finnish success, some of which has been surprising. The success in PISA is one of such type of success. One of the reasons of surprising can be related to the results of Finland in the International Mathematical Olympiads (IMO). Since the first time of participation in IMO in 1965, Finland has got only a modest success, especially in the years 1981, 1982 and 1983. The best result was that of 1982, when Finland got the 8th place among 23 countries. Inside Finland, there is agreement, between mathematicians, and even wider, about the level of school mathematics and level of students, where both are regarded as weak.

In 2005, at the conference "Teaching mathematics: beyond the PISA survey", organized by the Mathematical Societies of France and Finland, I gave a presentation entitled *"What are the Reasons Behind the Success of Finland in Pisa?"* A year later, the French Journal *Gazette des Mathématiciens* published a paper of mine with the same title (Malaty 2006). The same paper was published again at the end of 2006 by the Danish Mathematical Society, in a special issue of its journal *Matilde,* '*Mathematics in Finland*'. Upon an agreement between the Danish and Finnish Mathematical Societies, this paper was chosen to be the first with two commentaries. Both commentaries were written by well known mathematicians in Finland, and the first was signed by 107 mathematicians (Astela, et. al 2006, Kivelä 2006). Where no doubt, that all of the commentaries are representing facts about the weaknesses in the Finnish school mathematics, these facts do not contradict the other facts, represented by me about the reasons behind the success of Finland in PISA.
http://www.matilde.mathematics.dk/arkiv/M29/Matilde29.pdf

2. Between success and problems

From one hand we do deserve the success we have got in PISA, and from the other hand we do have serious problems in school mathematics.

2.1. How this could be explained?

PISA tests are measuring mathematics literacy. PISA test items are measuring the achievements of everyday life mathematics, including problems of no need to learn mathematics as a structure. We do know in Finland that we wouldn't get any success in PISA, if the test items were related to the understanding of mathematical concepts or relations. The most difficult to our students is to ask them to give a proof. This is understandable since school mathematics does not deal with mathematics as a structure. In the mentioned above paper, signed by 107 Finnish mathematicians, the authors demonstrate facts of Universities and Polytechnics students' mathematical knowledge decline (Astela, et. al 2006).

2.2. Curriculum changes and PISA

Before 1967, Finnish school curriculum was a traditional one, where among others students learn at secondary level Algebra and Geometry every week, where algebra had its own textbook and as well geometry. This was the case for about 100 years. Since 1967 school curriculum in Finland had seen different changes. These changes are mainly four: the New Math, especially from 1970 to 1980, the Back-to-Basics (1980-1985), Problem solving (1985-1990), and Everyday Life Mathematics (1990-1995). These trends are still effect on school mathematics in Finland, especially 'Everyday life Mathematics' and this effect has given a chance to success in PISA.

2.3. Why PISA is relevant and IMO not?

To get a success in IMO there is a need of taking care of gifted students, and this is not the case in Finland. On the other side, for more than 100 years, education has been provided for everybody upon some type of equality's principle. This has been strengthened in 1970 by the establishing of the Comprehensive School (Grades 1-9) as a compulsory education. This school forms what is called 'Basic Education'.

The Basic Education act of 1998 made clear that Basic Education has to provide each child with such knowledge and skills, which are necessary in everyday life. Also it puts emphases on the principle of equality between children's in education. Equality here has gained in Finnish society special meaning, which has effected on school mathematics and consequently on both PISA and IMO results. From one hand, mathematical curricula and textbooks have been built to be adequate to average students at maximum, and from the other hand, in each class, teachers have been active in recognizing students' weaknesses in time and offering soon remedial education, where also special teachers are available. To make this work possible, the number of students in a class has been relatively low. The majority of classes are of 15 to 25 students. Thus, the combination of school mathematics changes, since the Back-to-Basics, 'Everyday life Mathematics' in particular, and the principles of the Basic Education act related to the content and students' equality has given a relevant ground for the success in PISA, but not for the success in IMO. Here we can also mention to the fact that the time devoted to mathematics teaching in Finland is one of the lowest worldwide (UNESCO 1986). At the moment, we do have only 31 teaching hours per week, each of 45 minutes, for the 9 Grades of comprehensive school. This gives a mean of 2.6 hours per week for each grade, where an hour here is of 60 minutes. This low number of hours meets well with the objectives of the Basic Education act of 1998, where education is mainly for everyday life and equality is also between all school subjects. With this low number of hours it is difficult to success in IMO, but it is still possible in PISA with the limited objectives of 'Mathematics Literacy'. Below is a quotation from the Basic Education act 628/1998:

> *"Supporting pupils' growth towards humanity and ethically responsible membership of society, and to provide them with the knowledge and skills necessary in life… The instruction has to promote equality in society and pupils' abilities to participate in education and to otherwise develop themselves during their lives…"*

Here to mention that, at the time of the 'New Math', the time devoted to mathematics teaching at schools was much higher. This was not only because of the International effect and the Nordic one in particular, but also because of the press-up Finnish mentality in facing challenges. This gives us to understand, why we got the mentioned above success in the Mathematical Olympiads. It was the success of those students, who started theirs schooling in the years 1969, 1970 and 1971.

3. What are the reasons behind the success of Finland in PISA?

The mentioned above school mathematics changes, which have happened in Finland, have happened also in other countries, but why these changes have effected in the Finland's results in PISA more than in other places? One essential reason fact is the mentioned above care of students' weaknesses. The effect of this work was clear on PISA results, and without the care of students with learning difficulties we wouldn't get the first place in PISA in 2003. Nevertheless, this care wouldn't help alone to get the First place. There are six main reasons behind: 1) the success of pre-service teacher education, 2) the culture of the teaching profession, 3) the success of in-service teacher education, 4) the different efforts made to develop mathematics education, 5) the daily traditions of school life in Finland, 6) the continuity of teacher's work. Here, I am not going to give details about these reasons as the first five were discussed in an earlier paper mentioned above (Malaty 2006a, 2006b). Here I'll deal in brief with the basic strengths of pre-service teacher education and I'll discuss the sixths reason. About teacher training strengths and

weaknesses in Finland, I do have two papers, one of 2004 and the other of this year 2007; the first was modified for The National Board of Education of Finland
http://www.oph.fi/info/finlandinpisastudies/conference2005/malaty.doc.

In pre-service education, there are three aspects of strengths: a) keeping the level of teacher education qualification high, b) being able to recruit motivated students, c) providing teaching practice at University Practice Schools. In teachers work continuity there are two aspects of strengths: a) choosing teaching profession for life, b) rare changing of school.

Every teacher has to get a Master degree. It is M.Ed. in the case of both Primary School Teacher (Grades 1-6) and Special Teachers (Grades 1-9), and M.Sc. in the case of Secondary School Teacher (Grades 7-12).

Teaching training is one of the most popular fields of university applicants, especially Primary School Teacher Training, where we are able to recruit well-motivated students. Whereas we are able to recruit enough students to fill most of the places available for secondary mathematics teacher education, the number of applicants for primary teacher education is 5-8 times the number of places available. Those who fail to obtain a place normally apply again one or more times in the following years. It is also to notice that we do not have problems of teacher drop out. Those, who choose the teaching profession, are choosing it for life. One of the main reasons, for this aspect of strength, is the success in recruitment of motivated young people to teacher training. Here we need to put emphases in the fact that, salary is not the reason of young people interest in Primary Teacher Training. Indeed the salary is not bad, but on the other hand it is not enough high to be a motivation. The affective factor is a decisive one. Finnish youth remembers their time spent in Primary school, especially the early years, with great warmth. During these years, it is quite common to end the school day by shaking hands with the teacher and not uncommon to give the teacher a hug. This explains why the minor 'Teaching Beginners' is a popular choice of Primary School Teacher students. It also gives strength, especially in the case of Primary School, the rare changing of schools by teachers. Thus, teachers have a chance to develop their plans.

Providing teaching practice at University Practice Schools offers an ideal environment, where from one hand each trainee has the chance to get closed supervision as much as he likes from mathematics education specialists, and from the other hand all university facilities, including University Library are closed. In Finland, Teaching Practice schools are normally inside the university campus and closed to Teacher Training Departments, where mathematics education specialists are as well teaching practice tutors.

4. School Mathematics changes and oppositions

In Finland, mathematicians, among them Nevanlinna (Nevanlinna 1966) opposed the ‘New Math’ changes. The ‘Back-to Basics’, by its name hadn’t get at the beginning such opposition. Nevertheless, the level of universities and polytechnics students, after the disappearance of the effect of the New Math era, has made all the mathematicians in the country unhappy with the changes in school mathematics.

On the side of mathematics educators, most of them were involved in changes' activities. One of the leading figures of changes in the 1980s and 1990s was Erkki Pehkonen. In 1990, in a joined work with Bernd Zimmermann of Germany, Pehkonen declared that school mathematics is not mathematics, but an all-round educational subject, which is only called mathematics (Pehkonen and Zimmermann 1990, 10).

Despite the difficulty in being different, as a mathematics educator, I have been more closed to mathematicians view. I have been of the opinion that mental arithmetic, mechanical skill, problem-solving and everyday life mathematics can have a place in school mathematics, but they are not enough. One reason is that all these elements cannot give the needed base to higher education at universities and polytechnics. In addition, this would lead, at the end, to serious negative effect on the development of mathematical culture, and as well science and technology. The other reason is the need for every child to get formal experiences, which can allow him to enter the formal operational phase of Piaget. As the other formal science, i.e. logic, is not a school subject at the age of formal thinking development, mathematics is the only subject, which can offer the chance to every child to develop his/her formal thinking. Here we have to remember that there is obvious interplay between the individual issue and cultural issue. Cultural issue is in need of having individuals, who have the ability to continue the study at higher education.

5. Arithmetic teaching and the problems of learning algebra

Universities and Polytechnics mathematicians are sure of the weakness of today students in both algebra and geometry (Astela, et. al 2006, 9).

At the time of ‘Traditional Curricula’, before the ‘New Math’, geometry was the main way to develop students’ formal thinking. Today this is not the case, and we do have different problems in teaching geometry. Regarding the formal thinking development, geometry teaching has even negative effect on it. In this paper, we shall discuss in some details, only some aspects of the teaching of arithmetic and their relation to some of our problems in learning algebra.

In learning algebra, and since the beginning of the 1980s, we do face serious problems. One main reason here is the disappearance of the study, and even the use in intuitive way, of the properties of addition and multiplication operations, especially the properties of associativity and distributivity. This was done as one of the demands of reform, to make the "Back to basics" curricula different than that of the "New Math" ones. Taking away of everything related to the New Math era was a demand. Then, from the traditional curriculum, only skills, especially arithmetic ones, were brought back to be the core of school mathematics. The goal here was to face the critics of the New Math in declining children’s arithmetical skills. The way to achieve this goal is giving rules and drill children to use it to get correct answers.

First children learn to make drills in learning arithmetic, but this has continued to be also the way of learning algebra and geometry. In learning arithmetic, for addition, subtraction and multiplication of numbers, students learn to perform on a squared notebook and write these numbers one below the other. In the case of addition, children learn also to add more than two numbers in the same way, where the sign '+' has to be written only once preceding the last number and not between every two numbers. After getting the sum, the difference or the product, children have to write in a special line, and even inside a box the obtained number preceded by 'V:' 'V' is an abbreviation of the word 'Vastaus', i.e. 'Answer'. Similar procedures are also used in teaching 'long division'.

Squared notebooks are the only used in teaching and learning mathematics, even in solving a word problem or drawing a geometrical figure. Therefore, in Primary School tests, under each word problem a part of the test paper is squared. In some cases, children get zero mark in solving a word problem, because they weren't able to write their solution to the end in the given squared area. Here to notice that, it is common phenomena, when in word problem most students in a class get low points, but, going back to students notebooks, we find that these students were able to solve similar problems. The reason here is that textbooks give children the chance to drill themselves in solving similar word problems after the class discussion of an example. In the 1880s and 1990s, I had a chance to observe more than 2000 mathematical classes. It is remarkable to notice that, in a case of having a new foreign student, sometimes he/she was able to solve such problems and in some cases faster than Finnish students. The reason here was the economy of time by leaving the unreadable text and search for two numbers to perform in a similar way of the discussed example.

From the above discussion, we can notice that in Primary School (Grades 1-6) the use of the sign of equality '=' has been replaced by the use of 'V:' to mean answer. In the case of using the sign of equality, it is regarded also as 'answer'. This means that the sign '=' has lost its meaning. In addition, in textbooks, it is common to see squares, drawn to the right from the sign of equality. The number of these squares is the same, as the number of digits in the needed numeral. This means that the sign of equality has lost also its role. There is no place to write an equal expression and use the transitivity of equality. Thus Primary school years do not offer relevant ground to learn algebra.

Another problem of learning algebra is starting at Primary School. Instead of using the associativity of addition and multiplication, and the distributivity of multiplication over addition, textbooks since the Third Grade offer the so-called rule, or agreement, of the "Order of Operations", which they call 'Calculation Order'. This order is the same as in simple calculators and given in the next form: First calculate the inner brackets, Second multiply and divide, Third subtract from left to right. Students of Primary school, i.e. till reaching the age of 13 make drills in using this rule. This has brought to us a chronic problem, which we meet even with university students. To demonstrate this problem, let us take the next example: Simplify $9 + (1 + 5)$. For 20 years, I used to give this example to new Primary Teacher Training students. In every year not more than 5% have been able to use the associativity property, more than 50% apply the rule 'inner brackets first' and others took away brackets first, then added from left to right. I have got also similar results, when I gave the mentioned example to new Secondary Teacher

Training students. Also similar results I have gotten from giving the same example to teachers at in-service education. In all these cases, another problem was clear, and this is the fact that writing mathematical text correctly is a big problem.
It is here to mention that the using of the rule of "Order of Operation" is continuing till the end of Secondary School. One here may ask how then children can simplify an expression like $2x + 3y + 3x + y$, if students have to 'calculate' from left to right as they have learned? The textbook, which offers this example for the Students of Grade 7 (age 14), is using the next trick. First is given the next examples 2 apples + 3 apples = 5 apples, 2 *kg* + 3 *kg* = 5*kg*, $2m + 3\ m = 5m$, then the next statement is added $2x + 3x = 5x$, and finally the expression $2x + 3y + 3x + y$ where a number of apples is drawn to demonstrate the coefficient of x and a number of bananas is drawn to demonstrate the coefficient of y (Jaakola et. al 1995, 103-104). Here this trick helps in simplifying such expressions, where algebra is taught as 'the calculation of letters'. The textbook here gives students to understand that x is like apple, kilogram or meter to avoid the contradiction with the "Order of Operation" where the real nature of x is forgotten. The missing of studying the associativity and distributivity properties has changed the learning of algebra into another learning of mechanical arithmetic. Ignoring the trick mentioned above, in algebra students are mainly learning the substituting of letters by given numbers and again using the rule of "Order of Operation" to get right answers.

6. The future of school mathematics in Finland

In Finland, different efforts have been made to develop mathematics education (Malaty 2006a, 2006b). The results can be seen in terms of changes since 1995 towards building more balanced curriculum. From one hand we aim to keep our strengths in taking care of everyday life needs, but from the other hand we aim to build up mathematics as a structure. We have got some success in teaching mathematics in Senior Secondary School (Grades 10-12), and also in Primary School, especially Grades 1-2. The way to reach our goals is long and the process is slow. The phase, which needs more care, is that of Junior Secondary School (Grades 7-9). The most positive here is that the National Board of Education has published in 2004 the new 'Curriculum Basics', where 'mathematical thinking' and the 'structure of mathematics' are essential elements of the new curriculum. Our strengths have helped us in getting good results in teaching students with learning difficulties, and in the Success of PISA. These strengths can be a base for achieving also success in taking care of mathematics as a structure and assisting gifted students development.

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