ABSTRACT

In little over 15 years we have come from having just a stick of chalk - to a fabulous array of new classroom gadgets that those teachers who have received appropriate training can use to help to bring a very difficult subject to life. It can be argued that Mathematics now has more IT possibilities than any other subject, starting with Excel, and moving on to the bewildering and exploding array of web resources that includes Google Earth, YouTube videos, and a large number of beautiful java apps.

Then there is the use of amazing dynamic software such as Autograph, GSP and Cabri in a whole-class environment, but only if used effectively: how essential it is to engage the minds of the students before the software does anything.

So it is all worth it? What are the gains? What's just around the corner? Mathematics lessons can certainly be a lot more fun for both teacher and student, and that has to be good news. Hardware considerations will also be included in this presentation, part of which will be given from amongst the audience using a Bluetooth graphics tablet ....

INTRODUCTION

Most adults in the developed world now go to work in an environment that includes a broadband connected computer of some kind. Indeed, many find it almost impossible to work without this, and yet at the moment very few classrooms are similarly equipped, and this still remains a forlorn hope for many teachers.

A forward thinking school mathematics curriculum should therefore
- assume that a data projector and internet connected computer are available in the classroom
- assume that an internet connected computer are available for personal study
- acknowledge the phenomenal advance in dynamic software and computer algebra systems
- acknowledge the parallel explosion of web-based materials for mathematics teaching.
THE MODERN CLASSROOM
Ideally the projector should be ceiling mounted, so always available at the press of a button. The computer keyboard and mouse should be on a podium, so that they can be accessed while standing.

A mathematics classroom at Chu Van An National High School, Hanoi

How teachers, and pupils, interact with this screen can vary. The usual mouse and keyboard does not meet the important need to draw, which requires a pen. The main options for this are to use a Tablet laptop, or a Graphics Tablet:

The Tablet laptop, and the Graphics Tablet
THE SOFTWARE SCENE
This is moving ahead strongly with products such as:

- **Xthink** breaking the mould of interpreting hand-written mathematics
- **Geometry Expressions** breaking the mould of Constraint Geometry
- **Physics Illustrator** breaking the mould of self-animating diagrams
- **Cabri-3D** breaking the mould of 3D Euclidean Geometry
- **Autograph 3** breaking the mould of 3D coordinate Geometry (1)
- **TI N-Spire/Casio Classpad** breaking the mould of integrated CAS

Clever and fascinating as these products are, the fact remains that the great majority of high school mathematics teachers have yet to embrace the use of technology at all. This situation will only improve if the in-service training of teachers to use ICT becomes a far higher priority for Principals as well as Education Ministries, than it is at the moment.

BRINGING THE REAL WORLD MORE INTO THE CLASSROOM

Teachers who are fortunate enough to have a web-connected, projected computer image in their classroom have a “window on the world” that offers fantastic new opportunities.

**Having fun with Google Earth**

Google Earth (2) is freely available to anyone with a reasonably fast computer and internet connection, and offers the opportunity for an exciting new range of screen-based classroom activities. If you also download the Screen Protractor (3) you can easily measure angles of familiar objects around the world, for example the Pentagon in Washington.

![The Pentagon measured with an on-screen protractor](image)

![Average wind speed and direction over a period of time (4)](image)
Another resource is a web site which gives the average wind speed in many locations round the world. If you compare this with the Google Earth image of an airport runway you can confirm that the runway number is its direction in tens of degrees (eg “34” = 340°)

Using the Internet: The History of Mathematics

Mathematics has one of the richest histories of all school subjects, and a web-connected screen in the classroom gives the ideal opportunity to slip some of it seamlessly into teaching, without it seeming like a history lesson! For example a Pythagoras lesson can be greatly enriched by a visit to the TSM Resources site (5) where you can find out about the amazing Tunnel of Samos (6) that was constructed in Pythagoras’ time (and when was that? .. and where is Samos?).

The Samos tunnel was constructed in 550BC in consultation with Pythagoras, and they started from both ends. It can still be visited today!

MAKING SOME TRADITIONALLY DIFFICULT TOPICS APPEAR MORE STRAIGHTFORWARD

There are two basic influences of technology in the mathematics classroom: it can make a difference to how we teach, and it can also affect what we teach. The latter is a huge area that is being well aired elsewhere, but we now explore a few examples that have proved successful in bringing the subject to life in the classroom in ways which were impossible to imagine not so may years ago.
The mystery of calculus

The graphical approach offered by dynamic software these days can certainly give another layer of opportunity for tricky concepts to be understood. Being able to zoom in on 500 rectangles, under the curve $y = x^2$ from $x = 2$ to 4, illustrates a principle that can be very hard to get across by traditional means. Extend this to 3D and a whole new world opens up, making the study of volumes of revolution a natural follow-on to areas.

Autograph can now explore fully what’s going on when an area is rotated about an axis to form a volume.

The Mystery of Mathematics

Technology can certainly help to unravel some of the mystery of mathematics. For example the multifarious golden ratio can be explored in so many ways, including the human body, the Parthenon and a credit card! For inspiration try Gary Meisner’s Golden Number site (7).

Data from a classroom experiment to measure the ratio of height to belly-button, together with some other Golden Ratio favourites!
“I hate Vectors!”

Pupils in England have long found vectors troublesome, mainly because they could not easily visualise how they worked. However we now have dynamic tools which can illustrate vectors beautifully. Vectors need their own algebra, and this can be introduced very effectively by dragging points around the screen. As with areas and volumes, the extension to 3D is trivial, and even the ‘scary’ cross products can become de-mystified!

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**IMPORTANT MATHEMATICS TOPICS THAT SHOULD BE RE-INTRODUCED TO MAINSTREAM POST-16 TEACHING**

**Differential Equations**

Differential equations are usually regarded as the province of only the brightest in high schools. Once the principle of a rate of change of one variable with another has been grasped, together with Newton’s First Law, there is no reason why a graphical approach should not be used to bring this topic to life for even modest pupils.

Fascinating applications include falling out of an aeroplane, and predator-prey situations. If they can’t solve the equations they can certainly see what’s going on - and this is surely the equivalent to the numerical approach that is widely adopted by problem solvers in the ‘real’ world.

**Data, Data everywhere – the Central Limit Theorem**

The world, and the internet, is heaving with data. All the more important therefore to be able to manage it in the classroom, and inspire the young to take an interest in how data sets can be interpreted and how essential they are in everyday decision making.
The basis of a large proportion of data sets is the normal distribution. Why for example, on check-in at the airport, do they weigh our luggage, but not us? The best way to illustrate this is to use the Central Limit Theorem – a nasty theorem to prove, but perfectly understandable if you present it graphically.

![Graphs showing normal distribution of weights]

The normal distribution is everywhere! Here is the data from the 7500 patients at the Oundle surgery. On the left: male, female and children, and then all put together. The total weight of 250 random passengers turning up for a flight is very predictable!

**3D lines, planes, vectors and surfaces**

The world the students are learning about is 3D, so why not bring the study of 3D lines and planes back. It is very likely that this important topic was lifted up to the advanced levels because it is so difficult to visualise. Not so with the new generation of 3D software such as Autograph and Cabri.

The link with 2D can be enhanced:
- areas become volumes
- arcs become surfaces
- reflection in a line becomes reflection in a plane
- rotation about a point become rotation about a line
- vectors and matrices simply add a further element
CONCLUSION!
We have all watched the phenomenal growth in computer based mathematical technology over the past few years, but the time has surely come for a reassessment of what we teach in secondary mathematics and how we teach it. Up until now the only technology that could be assumed was handheld, and that too is continuing to make strong advances. With computer hardware becoming more affordable and the opportunities more engaging and exciting, there is a golden chance to save the subject from oblivion: using dynamic software and the internet can make the teaching far more effective and definitely more fun!

None of this of course can happen without appropriate training, and the TSM workshop model (8) at Oundle School, UK (a 3-day residential) is one solution that is now in its 6th year.

Douglas Butler, Oundle, June 2007

REFERENCES

(1) Autograph dynamic software       www.autograph-maths.com
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(6) The Tunnel of Samos             http://pr.caltech.edu/periodicals/EandS/articles/LXVII1/samos.html
(7) Gary Meisner’s Golden Number site http://goldennumber.net
(8) TSM Workshop (Oundle, UK) July 2007 www.tsm-resources.com/tsm-07

Douglas Butler: BIOGRAPHY (June 2007)
- after graduating in Mathematics and Electrical Sciences at Cambridge University, and a spell with EMI Records, Douglas has specialised in secondary Mathematics. He has served as Head of Mathematics at Oundle School (Peterborough UK), and was Chairman of the MEI Schools project, a leading UK curriculum development project, for 6 years.

A keen pianist and dinghy sailor, he is also author of "Using the Internet - Mathematics" (revised July 2003), the principal author of Autograph (version 3.2 June 2007), and a major contributor to "Teaching Secondary Mathematics with Technology" (Open University, October 2004). He maintains a large web site of educational resources in many subject areas.

In 2000 he founded the innovative iCT Training Centre, based at Oundle School, which is now creating new resources for the educational use of computers in mathematics, and running the TSM (Technology in Secondary/College Mathematics) teacher training events all over the UK and overseas. He is a frequent speaker at international mathematics teachers' conferences, and was the keynote speaker at the 2006 T³ Conference in Denver. He has also run a new series of conferences on Technology for Teaching Music.

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