Laplace transform to students of engineering
V.R.Lakshmi Gorty, Faculty in Mathematics
N.M.I.M.S University, M.P.S.T.M.E, Vile Parle (W), Mumbai lakshmi_gorty@rediffmail.com

Abstract:
The student’s outlook on fundamental ideas about Mathematics has changed to a greater extent. This paper is the case study of the students of engineering who have been given the syllabus of Laplace Transform and taught in the class the whole topic in the stipulated time duration. Applied Mathematics is very much important in Engineering subjects which forms the base for all others subjects. Students have come up with brighter ideas and majority of the students supported intensely the very old teaching techniques of mathematics.

Success is achieved in learning Laplace Transform with giving them the knowledge of the Mathematics tools available in the market. The seminars on this topic made the students expose themselves to in depth knowledge of Laplace Transform to the technical subjects. This gave an occasion for them to ask queries about the engineering subjects which are required for construction of an equation. Finally the basic need for Laplace Transform in Engineering could be understood.

Introduction:
The classroom methodologies used in Mathematics has been improved.
The students of Engineering speak out, when asked to them about the classroom teaching and the improvements required. These are the points laid stress on

1) More interactive
2) Length of lectures
3) Strength of the class
4) Question & answer session
5) Morning Lectures for the subject Mathematics
6) Practical in Mathematics
7) Convey the Knowledge
8) Tricks to learn the formulae
9) Software with Applied Mathematics
10) Exercises to be given in the class
11) Tutorials is a must
12) Continuous assessment is required
13) No matter how the ascent of the teacher is.
14) Seminar

1) Students of first year Engineering felt that the Applied Mathematics lectures must be more interactive than the a monotonous delivery of a lecture which can happen in other subjects where there are no much calculations .The student expects a query being asked after explaining each topic or a question to clarify the doubts of the students. The students say that a pause should be given to absorb the contents solved in the class so that they think themselves and would be ready for a similar question given question given next. A student says “a teacher can have a minute to look at a student, but the class should be made to look at the teacher for many such minutes, to define a good classroom condition”.

2) The length of the lectures for theory of Applied Mathematics should not exceed beyond two hours. In a lecture of speculated time they feel they are prepared to understand. The students feel that the too many hours assigned for the lecture would not benefit them. Instead they feel lot of tutorials should be introduced in the curriculum, which is at the moment there in their curriculum. The tutorial in the Applied Mathematics are been introduced where a student is given problems based on a particular type of problem done in a regular lecture. The Faculty prepares a question bank on the topic done in the last lecture and makes the students to do in tutorials /workshops. The students are very happy with this type of system where individual attention is paid. Here the time is not limited. The students are given four hours for a tutorial, which carries around fifteen to
twenty questions on the same topic. I have observed that a few students complete it very fast, may be within two hours and others extend beyond four hours also.

3) Students feel that depending on the **strength of the class** the lecture in Mathematics must be conducted; otherwise they sometimes miss the topics, which have a wide use in the engineering subjects.

4) The **question and answer session** is a must after every lecture. The students who were in the class can judge themselves of how much they have understood. Also it is a check for the teacher whether he/she has conveyed the contents properly. The students of different engineering backgrounds should be clarified about their doubts that they are going to apply in mathematics.

5) It is observed that Mathematics needs lot of concentration and energy to solve problem the problems. So the students say that they prefer **morning lectures** for all applied Mathematics. When the students are given the afternoon lectures that is after lunch session they feel sleepy and could not take it easy. These topics are to be paid utmost importance as it stands as the base for all engineering subjects. The faculties also feel that the content per topic delivered is received at a higher efficiency in the morning hours than in the afternoon sessions. Apart from all this, the morning lectures make a good start with mathematics.

6) I personally feel that some **practical sessions** would make a difference in understanding the subject. In the sense the faculty of engineering subjects can be asked to set up equipment, which may give the direction of Kirchhoff’s voltage law, Simple harmonic motions, LCR circuits and many such applications, which can be done, in a practical room. Then the Mathematics faculty can be asked to develop the equations or solve the applications with reference to the mathematics, which they learnt during their regular lectures. This can encourage the interdisciplinary sciences as well as the importance of applied mathematics can be noticed then and there.

7) The basic mathematics is expected from a student of engineering who has scored high in his secondary school level and thus got into engineering. Still it observed that many students tend to forget the basic integrations and differentiations. So when I start with the lecture in mathematics, to start with Laplace Transform, I ask the students to write the formulae of the basic integrations on a separate sheet of paper. The students are presupposed to bring this sheet for every lecture of mathematics to update the new formulae given to them relating to the Laplace transform. Thus by the end of the topic Laplace Transform, the student not only framed all the required formulae for this chapter on a sheet, but also by now remembers all the applied mathematics formulae required to apply to the questions based on it. To **convey the knowledge**, I feel repeated practice also is an important tool which makes a student perfect.

8) The major hurdle the students face is to remember all the formulae to solve the problem successfully. They asked me a method to remember all the formulae. The above method was one of the methods adapted, which was a success to a major extent. Another method was to write all the formulae on the board simultaneously again and again for the whole class. This was more of spoon-feeding technique, which can be avoided for the class of brighter students. **Tricks to learn formulae** every now and then can be mentioned by means of some series or songs or tunes. For example: For calculating Laplace Transforms of a sine function, the value is given with the coefficient of t’, “sine carries its sign”.

9) Now the Information Technology has entered into every aspect of Engineering. When I asked to my students what, if I give them the knowledge of the Mathematics tools available in the market? The students said that they wouldn’t mind to be introduced for the knowledge sake, but they would like to learn mathematics only on the board with chalk and duster. The age old teaching techniques are the best to learn mathematics. I conveyed them the tools of Mathematics like MATHEMATICA, MATLAB, MAPLE and many others likewise. They said that at the end of a lecture they could be introduced to such tools. The students felt that such ready-made tools would spoil their basic learning Applied Mathematics. **Ready software** to solve Laplace Transforms should be not very much in the curriculum.

10) Applications or word problems are to be given as **exercises** to the students so that not only the mathematical problems but also they can be exposed to the application problems. The preference
to such problems was given as a homework session. Thus the wide covering of the particular topic can be done.

11) A tutorial is a must as majority of students have voted for it. In the tutorials the students are supposed to write the answers to the given set of problems. They are free to ask any doubts during their solving. I personally go around checking the method by which an individual is trying. Sometimes I guide them to change the track when I feel the student is in a wrong way solving the problem. During these sessions it is taken utter care of an individual for making them comfortable with applied mathematics as well as fear about the subject is relieved by a natural way of practicing. The students try to compete with their classmates thus making the students active. Also the slow students try to make a stand of where they are with respect to the class without actually, me making any efforts to convey the message.

12) One after the other, the assignments keeps a student in the form. With the semester/trimester system of courses continuous assessment keeps them on the toe. By continuous assessment I mean, taking regular tests, giving homework, exercises and tutorials. The continuous assessment may also include oral exams in between the tutorial sessions, which can be based on the revision of the basics of mathematics.

13) The students say that the ascent of the teacher doesn’t affect the classroom environment for learning mathematics. Perhaps the teacher should be intelligent and knowledgeable in the subject mathematics.

14) Finally after completing the lesson, I asked the students to prepare for a presentation on one of the application questions from Erwin Kreyszig (a book on Higher engineering Mathematics). For the presentation they went around the faculty of core engineering and clarified about the technical aspect of the problem. Then they came to me to find how it is derived mathematically. Finally combined with all the ideas from engineering as well as mathematics they have delivered their ideas in the form of a seminar. By this I felt that a complete conversion of applied mathematics to engineering is been achieved. The students answered contentedly when they were asked about how they feel.

The final review of the lesson was taken and found that students have understood completely of how to go with applied mathematics to take a long way to engineering. The students are exposed to different environments of learning and are skilled in the way they are able to receive the subject. The students nearly discouraged the use of projector or any mode of classroom technologies for the subject applied mathematics.

The students performed better after the case has been observed and being implemented all the techniques given above. Finally a success was achieved in making the students solve the problems perfectly.

The case study of the students during the course of teaching Laplace Transform has been given here. Initially they try to attempt the formulae given to them without actually understanding the properties but later get to know the perfect method by practice.

Case study done by V.R.Laksmi Gorty for the engineering students in Laplace Transform

Q.1) Find Laplace Transform of $f(t)$ where

$$f(t) = \cos(t - \pi)$$

$= 0$  [t<]

Answers as per the understanding of the student in the classroom

**Student (A) By second shifting property**

If $f(t-a) = \begin{cases} f(t); & t > a \\ 0; & t < a \end{cases}$ then $L \{f(t-a)\} = e^{-as}L\{f(t)\}$

$L \{\cos(t-\pi)\} = e^{-s^2} \cdot \frac{s}{s^2 + 1}$
Student (B) \( L\{\cos(t-\pi)\} = \int_{0}^{\infty} e^{-st} \cos(t-\pi)dt \)
\[ = \int_{0}^{\pi} e^{-st} \cos(t-\pi)dt + \int_{\pi}^{\infty} e^{-st} \cos(t-\pi)dt \]
\[ = 0 + \int_{\pi}^{\infty} e^{-st} \cos(t-\pi)dt \]
\[ = \int_{\pi}^{\infty} e^{-st} \cos(t-\pi)dt \]

Student (C) \( L\{\cos(t-\pi)\} = e^{s(t-\pi)} \cos t \)

Student (D) \( L\{\cos(t-\pi)\} = L\{\cos t \cos + \sin t \sin \} \)
\[ = L\{-\cos t\} \]
\[ = \frac{1}{s^2 - 1} \]

Q.2) Solve the inverse Laplace Transform of
\[ \frac{1}{(s^2 + 2s + 5)(s^2 + 2s + 2)} \]

Answers as per the understanding of the student in the classroom

Student (A) \( L^{-1}\left(\frac{1}{(s^2 + 2s + 5)(s^2 + 2s + 2)}\right) \)
\[ = L^{-1}\left(-\frac{1}{3} \left(\frac{1}{s^2 + 2s + 5} - \frac{1}{s^2 + 2s + 2}\right)\right) \]
\[ = -\frac{1}{3} L^{-1}\left(\frac{1}{(s + 1)^2 + 2^2}\right) + \frac{1}{3} L^{-1}\left(\frac{1}{(s + 1)^2 + 1}\right) \]

Student (B) \( L^{-1}\left(\frac{1}{(s^2 + 2s + 5)(s^2 + 2s + 2)}\right) \)
\[ = L^{-1}\left[\frac{A\cdot s + B}{s^2 + 2s + 5} + \frac{C\cdot s + D}{s^2 + 2s + 2}\right] \]
with partial fraction method, solve for A, B, C, D

Student (C) \( L^{-1}\left(\frac{1}{(s^2 + 2s + 5)(s^2 + 2s + 2)}\right) \)
\[ = L^{-1}\left(\frac{1}{s^2 + 2s + 5}\right) L^{-1}\left(\frac{1}{s^2 + 2s + 2}\right) \]
\[ = L^{-1}\left(\frac{1}{(s + 1)^2 + 2^2}\right) L^{-1}\left(\frac{1}{(s + 1)^2 + 1}\right) \]
\[ \begin{align*}
\text{Student (D)} & \quad \text{L}^{-1} \left( \frac{1}{(s^2 + 2s + 5)(s^2 + 2s + 2)} \right) \\
& = \text{L}^{-1} \left( \frac{1}{(s + 1)^2 + 2^2 \left( (s + 1)^2 + 1 \right)} \right) \\
& = e^{-t} \sin 2t \quad \frac{1}{2} \left( \frac{1}{(s + 1)^2 + 1} \right)
\end{align*} \]

Q.3) Q.2) Solve the differential equation with Laplace Transform
\[ \frac{d^2y}{dt^2} + 9y = \cos t, \quad y(0) = 1, \quad y(\pi) = 0 \]

Answers as per the understanding of the student in the classroom

**Student (A)** Applying Laplace Transform to the above equation, it becomes
\[ s^2 \overline{y(s)} - sy(0) - y'(0) + 9 \overline{y(s)} = \frac{s}{s^2 + 1} \]

substituting \( y(0) = 1 \) and \( y'(0) = k \) (constant)

we get,
\[ s^2 \overline{y(s)} - s(1) - k + 9 \overline{y(s)} = \frac{s}{s^2 + 1} \]

**Student (B)** Applying Laplace Transform to the above equation, it becomes
\[ s^2 \overline{y(s)} - sy(0) - y'(0) + 9 \overline{y(s)} = \frac{s}{s^2 + 1} \]

substituting \( y(0) = 1 \) and \( y'(0) = 0 \)

we get,
\[ s^2 \overline{y(s)} - s(1) - 0 + 9 \overline{y(s)} = \frac{s}{s^2 + 1} \]

**Student (C)** Applying Laplace Transform to the above equation, it becomes
\[ s^2 \overline{y(s)} - sy(0) - y'(0) + 9 \overline{y(s)} = \frac{s}{s^2 + 1} \]

**Student (D)** since \( y'(0) \) is not given this problem cannot be solved.

**Observation:** I realize that even if I have taken all the several methods of solving problem techniques of Laplace Transform, there are few students who carelessly or illogically commit mistakes. Exceptions are always there. If we leave out those one percent cases then I have put a step towards achieving success in making the engineering students to apply mathematics in their engineering branches.