# Curriculum Vitae


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## Positions

- April 2007 – present, Researcher, Department of Mathematics and Applications, University of Palermo, Italy.
- October 2006 – April 2007, Postdoctoral fellow, Department of Mathematics, University of Bologna, Italy.
- November 2005 – October 2006, Research fellow, Department of Mathematics and Applications for the Architecture, University of Firenze, Italy.

## Education

- Degree in Mathematics. University of Florence, Italy.
  Title of the thesis: Propagation of Electromagnetic Waves in optical waveguides with circular profile. Advisor: Prof. Rolando Magnanini
- Ph.D., Mathematics, University of Firenze, Italy. January 2003 - April 2006.
  Title of the thesis: Non-rectilinear waveguides: analytical and numerical results based on the Green's function. Advisor: Prof. Rolando Magnanini

## Visiting positions

- Institute of Mathematics and its Applications, Minneapolis, August 2004.
- Institute of Mathematics and its Applications, Minneapolis, September - December, 2005.
- Department of Mathematics, Graduate School of Science and Engineering of Ehime University, Matsuyama (Japan), February 2007.
- Centre de Mathématiques Appliquées, Ècole Polytechnique, Paris, France, October 2008 - February 2009.
- Department of Mathematics, Inha University, South Korea, December 2008.

## Teaching

- Analysis I (Prof. G. Modica), Engineering Faculty of Firenze, 2003.
- Integral Calculus (Prof. R. Magnanini), Computer Science Faculty of Firenze, 2005.
- Institutions of Mathematics II (Prof. A. Cianchi), Faculty di Architecture of Firenze (AA 2005/2006).
- Institutions of Mathematics I (Prof. P. Gronchi), Faculty di Architecture of Firenze (AA 2005/2006).
- Differential Calculus (Prof. E. Francini), Computer Science Faculty of Firenze, (AA 2006/2007).
- Institutions of Mathematics I (Prof. A. Cianchi), Faculty di Architecture of Firenze (AA 2006/2007).
- Mathematical Analysis II, Physics, University of Palermo (AA 2007/2008).
- Mathematical Analysis II, Physics, University of Palermo (AA 2008/2009).

## Talks

- "Wave propagation in optical fibers", Dipartimento di matematica Ulisse Dini, Firenze, March 2004.
- "Surface Registration Via Umbilical Points", IAC, Firenze, October 2004.
- "Propagazione ondosa in fibre ottiche" - Workshop su "Equazioni a derivate parziali: aspetti metodologici, modellistica, applicazioni", Ragusa Ibla, June 2005.
- "Wave propagation in optical waveguides. Numerical results." - 5th International ISAAC Congress, Catania, July 2005.
- "Wave propagation in optical waveguides with imperfections", workshop "Imaging from Wave Propagation" Poster session, Institute for Mathematics and its Applications (IMA), Minneapolis (USA), October 2005.
- "Non-rectilinear optical waveguides: analytical and numerical results based on the Green's function", Department of Mathematics of Minneapolis, University of Minnesota (USA), November 2005.
- "Wave propagation in optical waveguides: analytical and numerical results based on the Green's function and a radiation condition for uniqueness", Ehime University, Matsuyama (Japan), February 2007.

- "Condizione di radiazione e unicità per la propagazione guidata di onde", Dipartimento di Matematica "U. Dini", Firenze, March 2007.
- "Condizione di radiazione e unicità per la propagazione guidata di onde", XVIII Congresso UMI, Bari, September 2007.
- "Wave propagation in optical waveguides: a radiation condition for uniqueness of solutions", Institut Henri Poincar, Paris, October 2008.
- "Wave propagation in open waveguides: a mathematical analysis based on the knowledge of a Green's function", Inha University, Incheon (South Korea), December 2008.

# Publications

- Oleg Alexandrov – Giulio Ciraolo, *Wave propagation in a 3-D optical waveguide*, Mathematical Models and Methods in Applied Sciences (M3AS), 14 (2004), no.6, 819–852.
- Oleg Alexandrov – Giulio Ciraolo, *Wave propagation in a 3-D optical waveguide II. Numerical Results*, to appear in "Continued Progress in Analysis, Proceedings of the 5th International ISAAC Congress", World Scientific.
- Giulio Ciraolo, *Wave propagation in non-rectilinear waveguides*, Le Matematiche, Fascicolo II, Vol LX (2005), 445–450.
- IMA report: Thomas Grandine, Jung-ha An, Viktoria Averina, Wondimagegnehu Geremew, Derek Hansen, Guo Luo, Todd Moeller, *Surface Registration via Umbilics*.
  http://www.ima.umn.edu/modeling/mm04.html.
- Giulio Ciraolo, *Non-rectilinear waveguides: analytical and numerical results based on the Green's function*, Ph.D. Thesis.
- Giulio Ciraolo – Rolando Magnanini, *Analytical results for 2-D non-rectilinear waveguides based on the Green's function*, Math. Methods Appl. Sci., 31 (2008), no.13, pp. 1587-1606.
- Giulio Ciraolo – Rolando Magnanini, *A radiation condition for uniqueness in a wave propagation problem for 2-D open waveguides*, accepted by Mathematical Methods in the Applied Sciences.
- Giulio Ciraolo, *A boundary variation method for waveguide couplers*, Applicable Analysis, 87 (2008), no.9, 1019-1040.
- Giulio Ciraolo, *Uniqueness of a solution for wave propagation problems with guided waves*, preprint.

# Research

My research concerns the study of wave propagation in open optical waveguides.

The problem under consideration is that of existence and uniqueness of solutions to the scalar Helmholtz equation. The starting point is a work by Magnanini and Santosa [MS], where the authors found a Green's function for the equation

$$\Delta u + k^2 n(x)^2 u = f(x,z), \quad (x,z) \in \mathbb{R}^2, \tag{1}$$

where the index of refraction $n$ is of the form

$$n(x) = \begin{cases} n_{co}(x), & |x| \le h, \\ n_{cl}, & |x| > h. \end{cases}$$

With such a choice of $n$, $(1)$ models the electromagnetic wave propagation in 2-D infinite and open rectilinear optical waveguides (the axis of the waveguide is assumed to coincide with the $z$-axis).

The problem can be analyzed by looking for solutions of the homogeneous Helmholtz equation of the form $v(x,\beta)e^{ik\beta z}$ and thus by studying the associated eigenvalue problem

$$v'' + k^2[n(x)^2 - \beta^2]v = 0,$$

with the condition that $v(x)$ is bounded (here $\beta$ is the spectral parameter).

Under our hypothesis on $n$, it results that the spectrum consists of discrete and continuous parts.

Such an eigenvalue problem (and the consequent Green's function) emphasizes clearly the different kinds of energy which may appear in the presence of an optical waveguide. In fact, the Green's function $G$ obtained in [MS] can be written as

$$G = G^g + G^r + G^e.$$

Here, $G^g$ is the part of the Green's function which describes the guided energy, which propagates mostly inside the core of the waveguide as a finite number of guided modes and corresponds to the discrete part of the spectrum. On the other side, the continuous spectrum corresponds to $G^r + G^e$, i.e. the energy radiated outside the waveguide. $G^r$ and $G^e$ correspond to radiation and evanescent modes, respectively, according to their different behaviour along the axes.

Starting from such results, in my PhD Thesis and following works I studied the problems listed below.

## 3-D waveguides: optical fibers.

In [AC1], the results in [MS] are generalized to the case of a 3-D cylindrical optical fiber and related numerical examples are presented in [AC2]. In these works, the Green's function is obtained by using a different technique with respect of the one adopted in [MS]. In particular, due to a different kind of associated eigenvalue problem, we use the Weyl-Titchmarsh theory on singular differential operators.

## Non-rectilinear waveguides: analytical and numerical results.

In [CM1], we propose a mathematical framework which allows us to study the field in 2-D waveguides which are "small" perturbations of rectilinear ones.

By changing the coordinates, we transform the non-rectilinear waveguide in a rectilinear one; in the new coordinates, the index of refraction $n$ depends on both the $x$ and $z$ variables. By using the Green's function obtained for the rectilinear configuration, an existence theorem for the perturbed problem is established by using a fix-point argument.

In [Ci2], we prove that the Neumann series associated to such a fix-point scheme converges and we apply the analytical results in [CM1] to relevant cases for the applications of optical waveguides by showing some numerical results.

## Uniqueness of solutions and radiation condition.

When studying the Helmholtz equation in the free-space (constant index of refraction), the assumption which guarantees the uniqueness of solutions is the well-know *Sommerfeld radiation condition*. This condition describes the behaviour of the wave far from the source and implies that the solution behaves as a spherical wave which propagates towards infinity. Such a condition has been extended by many authors to cases in which the index of refraction is constant outside a bounded set or, more in general, when the index of refraction has a suitable behaviour at infinity.

In the case of an open waveguide, the classical radiation condition does not apply to our problem because the inhomogeneity of the index of refraction extends to infinity in one direction. Also, because of the presence of a waveguide, some waves propagate in one direction with different propagation constants and without decaying in amplitude.

In [CM2], we provide an explicit condition for uniqueness which takes into account the physically significant components, corresponding to guided and non-guided waves; this condition reduces to the classical Sommerfeld-Rellich condition in the relevant cases. In the same paper, by careful asymptotic estimates, we also show that our condition is satisfied by the solution found in [MS].

The results in [CM2] are used in [Ci2] and [Ci3] to prove the uniqueness for non-rectilinear waveguides.

## Future work

The extension to 3-D waveguides of the results obtained for the 2-D case is one of the goals of my future research. The starting point are the results already obtained in [AC1].

The radiation condition given in [CM2] is important not only for theoretical reasons but also for numerical aspects of wave propagation.

Numerical computations based on the Green's function are not satisfactory, since they require a large numerical effort and do not produce precise enough simulations (one has to deals with infinite integrals of oscillating functions). To avoid these problems, many numerical methods have been proposed for the Helmholtz equation (without the presence of a waveguide). The main idea is to define an artificial computational domain and prescribe the boundary conditions which best approximate the problem over the whole space ($transparent\ boundary\ conditions$). We cite [Be], [EM] and [KG] as fundamental works on this topic. The results on the uniqueness found in [CM2] can be a starting point for studying the problem of transparent boundary conditions for waveguides problems.

# References


[AC1] O. Alexandrov – G. Ciraolo, *Wave propagation in a 3-D optical waveguide*. Math. Models Methods Appl. Sci. (M3AS) 14 (2004), no. 6, 819–852.

[AC2] O. Alexandrov e G. Ciraolo, *Wave propagation in a 3-D optical waveguide II. Numerical results.* To appear in "Continued Progress in Analysis, Proceedings of the 5th International ISAAC Congress", World Scientific.

[Be] J.P. Berenger, *A perfectly matched layer for the absorption of electromagnetic waves*. J. Comput. Phys. 114 (1994), no. 2, 185–200.

[Ci1] G. Ciraolo, *Non-rectilinear waveguides: analytical and numerical results based on the Green's function*, PhD Thesis.

[Ci2] G. Ciraolo, *A method of variation of boundaries for waveguide grating couplers*, preprint.

[Ci3] G. Ciraolo, *Uniqueness of a solution for wave propagation problems with guided waves*, in preparation.

[CM1] G. Ciraolo – R. Magnanini, *Analytical results for 2-D non-rectilinear waveguides based on the Green's function*, accepted for publication in Mathematical Methods in the Applied Sciences.

[CM2] G. Ciraolo – R. Magnanini, *A radiation condition for uniqueness in a wave propagation problem for 2-D open waveguides*, submitted.

[EM] B. Engquist – A. Majda *Absorbing boundary conditions for the numerical simulation of waves*, Math. Comp. 31 (1977), no. 139, 629–651.

[KG] J. B. Keller – D. Givoli, *Exact nonreflecting boundary conditions*, J. Comput. Phys. 82 (1989), no. 1, 172–192.

[MS] R. Magnanini and F. Santosa, *Wave propagation in a 2-D optical waveguide*, *SIAM J. Appl. Math.,* **61** (2001) 1237 – 1252.

[Re] F. Rellich, *Über das asymptotische Verhalten del Lösungen von $\Delta u + \lambda u = 0$ in unendlichen Gebieten*, Jahresbericht der Deutschen Mathematiker-Vereinigung, 53 (1943), pp. 57 – 65.