OPEN-ENDED APPROACH TO TEACHING AND LEARNING OF HIGH SCHOOL MATHEMATICS.
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Abstract
The author shares some of the findings of the research he conducted in 2007 on grade 11 mathematics learners in two schools, one experimental and the other one control. In his study, the author claims that an open-ended approach towards teaching and learning of mathematics enhances understanding of mathematics by the learners. The outcomes of the study can be summarised as follows:

1. In the experimental school, where the author intervened by introducing an open-ended approach to teaching mathematics (by means of giving the learners an open-ended approach compliant worksheet to work on throughout the intervention period), the performance of the learners in the post-test was better than that of the learners from the control school. Both schools were of similar performance in the pre-test. The two schools wrote the same pre-test and same post-test. Both schools were following common work schedule.

2. Within the experimental school, post-test performance of the learners in the class where the intervention was monitored throughout the intervention period (thus ensuring compliance of the teacher to the open-ended approach) out-performed those in which monitoring was less frequent.

3. There was no significant difference in performance between learners from the unmonitored experimental class and those from the control class.

1. Introduction
The kind of teacher envisaged by the New Curriculum Statement (NCS), the curriculum followed in South Africa, includes, among others, being a mediator of learning, and a designer of Learning Programmes and material (DoE, 2002:3). The outcomes specified in the NCS encourage a learner-centred and activity-based approach to education (DoE, 2002:1). According to the Department of Education (DoE) (2008:10), Education and Training in South Africa has 7 critical outcomes and 5 developmental outcomes, which derive from the Constitution. Each of them describes an essential characteristic of the type of South African citizen the education sector hopes to produce. The document further states that these critical outcomes should be reflected in the teaching approaches and methodologies that mathematics teachers use.

The above discussion possibilitated the checklist as reflected on the table below. According to DoE (2005:8), for instance, ‘all learners in Grades 10 – 12 should be given the opportunity of developing themselves mathematically’.

To measure how well a student performs, teachers have to be able to examine the process of learning, not just the final product (Badger and Thomas, 1992). Such a view of learning and teaching demands an “open-ended” form of teaching, learning and assessment, based on open-ended tasks and questions (Moschkovich, 2004:51-53; Radford, 2001:251; Elbers, 2003:91; Hershkowitz & Schwarz, 1999:150). In her detailed analysis of two United Kingdom schools, Boaler (1997) argues that the school using an open approach to teaching and learning mathematics produced more sustained outcomes in mathematics learning, than the conventional format used by the other school.

The open-ended questions asked are the type of questions asked in a socio-constructivist lesson. The solution path of the learner, rather than that of the teacher, governs the teacher’s intervention.
<table>
<thead>
<tr>
<th>Critical Outcome</th>
<th>The teacher’s approach:</th>
<th>Yes</th>
<th>No</th>
</tr>
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<tbody>
<tr>
<td>1. Identify and solve problems and make decisions using critical and creative thinking.</td>
<td>1. Makes it possible for learners to have opportunities to make comprehensive use of their mathematical knowledge and skills.</td>
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<td>2. Work effectively with others as members of a team, group, organisation and community</td>
<td>2. Encourages an active learner participation in lessons and allow the learners to express their ideas more frequently.</td>
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<td>3. Organize and manage themselves and their activities responsibly and effectively.</td>
<td>3. a) Provides every learner with a reasoning experience  b) Positions the teacher as the facilitator, and not the source, of learning.</td>
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<tr>
<td>4. Communicate effectively using visual, symbolic, and/or language skills in various modes.</td>
<td>4. Makes it possible for every learner to respond to the problem in some significant ways of his/her own.</td>
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2. Research Design:
The approach used was intended to answer the following research question: “What will be the impact of an open-ended approach on the learning of mathematics in grade 11 mathematics classes at the selected experimental school?” [The full report is a PhD thesis that has just been submitted. For further details consult my promoter, Prof HD Nieuwoudt, at hercules.nieuwoudt@nwu.ac.za.] 
The design of the complete research was a mixed-method approach adapted from Creswell (2003). This report focuses only on the Phase 1 component of the study.

Phase 1 of the investigation was the quantitative part of the investigation where results from experimental and control schools were compared to explore impact of open-ended approach on learning of mathematics. Learners from the experimental school and control school wrote a pre-test at the beginning of the study to establish and compare their pre-requisite knowledge. Analysis of the pre-test results (to be presented at the conference) showed that there was no significant difference in performance between the three categories of learners – the monitored experimental group, the unmonitored experimental group as well as the control group.

The author prepared and gave learners from the experimental school a worksheet covering mathematics topics the two schools – experimental and control – followed. The worksheet asked predominantly open-ended questions and/or tasks for learners to solve. Though questions asked were open-ended, grade 11 mathematics topics used were the traditional topics. The worksheet (also to be shown at the conference) was designed to be used by the learners to solve mathematical problems instead of the learners having to be taught by the teacher from the classroom front. The teacher’s role was only to facilitate the learners’ attempts to solve the problem. The choice of the topics was based on the common work schedules both schools used. The learners from both experimental and control schools wrote a post-test at the end of the intervention period, compiled by the author in consultation with the teachers from both schools. The purpose of the post-test was to see if there was any post-intervention difference in performance between the two groups. All the classes that formed part of study covered the same work schedule and wrote the same tests. Care was taken by the research design to limit other possible factors (some of which will be mentioned at the conference) accounting for different behaviours of the groups as far as performance in the tests is concerned.

Regular monitoring of the implementation of the open-ended approach to teaching and learning was done on two of the four experimental classes. The other two were only supplied with the
worksheet to use throughout the period of intervention. The design of the investigation was such that the monitored class was the only one in which the teacher’s compliance to the open-ended approach was adequately monitored by the author. Because of lack of consistent monitoring on the other classes in the experimental school, there was not enough evidence of compliance or otherwise to the open-ended approach to teaching and learning in those classes. However, a video of the unmonitored class, recorded during the intervention period, pointed to, among others, passiveness among other learners while a volunteer learner attempted to solve the problem on the chalkboard. The video of the control school, also taken during the intervention period, showed teacher approach that definitely did not comply with the open-ended approach to teaching and learning. The main focus of monitoring in the two classes was to establish if the type of questions the teacher was asking the learners during the solution process complied with the expectations of the open-ended approach to teaching and learning. The initial briefing of the experimental school teachers by the author at the beginning of the study was intended to explain to the teachers exactly what constituted compliance to the open-ended approach to teaching. However, it subsequently came out that some of the proceedings in the unmonitored classes were not fully compliant to the approach.

3. Phase 1 Results: Pre-test – Post-test

The t-test was used to compare the groups. The difference, if any, of the means of the compared groups was considered to be significant if the significance level \( p \) was at most 0.05 \( (p \leq 0.05) \). In cases where significant differences were obtained, the effect size \( d \) was calculated to establish the practical significance of the result (Steyn, 2009).

3.1. Pre-test results

The aim of the pre-test was to establish the prerequisite knowledge of the learners. The prerequisite knowledge mentioned here was the mathematical knowledge required to facilitate the learners’ understanding of the mathematical topics covered during the period of intervention.

3.1.1. Conclusion: Pre-test

There was no significant difference in performance between each pair of the three groups: unmonitored experimental \( (N = 73) \), monitored experimental \( (N = 93) \) and control \( (N = 88) \), as far as their pre-requisite knowledge is concerned. This seems to justify the conclusion that the groups the study investigated were of comparable pre-requisite knowledge.

3.2. Post-test results (These will be presented at the conference)

There were two contexts of looking at the post-test results. The first one was in terms of comparing averages of the post-test marks themselves, while the second was in terms of looking at the post-test marks on a question-by-question basis. This report only deals with the post-test averages.

3.2.1. Average post-test performance: Conclusion.

The monitored group outperformed both the control group and the unmonitored experimental group as far as average performance in the post-test was concerned. However, there was no statistical difference in performance between the unmonitored group and the control group.

3.4. Conclusions: Pre-test and Post-test phase

In general, the pre-test – post-test data showed that the monitored group outperformed both the unmonitored group and the control group in the post-intervention test. There was, however, no significant difference between the unmonitored experimental group and control group. This is despite the fact that in the unmonitored experimental school the open-ended approach compliant worksheet was used. With all things – prerequisite knowledge, similar school resource environment, etc. - being the same, one can attribute the difference in results to the role the teacher played during the intervention. If one compares the monitored and unmonitored classes, one realises that both were of similar pre-requisite knowledge, both were of the same school, and both were using the same intervention material. The main difference was in terms of the approaches adopted by the teachers in both classes. Improved performance in the post-test
results for the monitored experimental group prompts one to agree with the statement of Hiebert et al. (1996) that when a student learns mathematics through such a problem-based approach, struggling with the difficulties facing him instead of relying on memorisation or any pre-determined rule to search for solutions, it promotes “deep understanding” of the mathematics that is valued. Hodgson and Watland (2004:1), in talking about OEA, said: “Through groups and other learning interactions with their online peers, students acquire deeper understanding because of the opportunities for exposure to multiple perspectives and interpretations”.

Mewborn, Lawrence and Leatham (2005:416) also had a positive comment to make about the open-ended approach to teaching and learning:

“I have noted significant improvement in my students’ self-confidence and their willingness to share their thinking with others. In fact, they begin to take pride in their explanations and find satisfaction in being able to explain what they are doing and why. They begin to see that there is a point to explaining their thinking. This leads to students feeling more ownership of their mathematical learning”

References

HODGSON, V. & WATLAND, P. 2004. The social constructionist case for researching networked management learning: A postscript and reply to Arbaugh and Benbunan-Fich in Management Learning. Vol. 35(2)